MATHEMATICAL ANALYSIS FOR A NEW TENNIS BALL LAUNCHER

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Abstract: The paper presents the mathematical analysis for the design of a new tennis ball launcher in order to assess the possibilities for its technical implementation. First, traditional launchers are described. Next, several new requirements improving training possibilities of such machines are suggested. The motion equation of the flying tennis ball is formulated and numerically solved. This makes it possible to analyze the trajectories of the ball for different initial conditions: elevation and heading angles, as well as the rollers angle. Then, the mathematical analysis of the launcher with two counter rotating rollers is presented. Stiffness (Young's) modulus and friction coefficients for the typical tennis ball have been determined experimentally. Based on these, initial conditions for the throw have been found: rotating speeds of the rollers and powers of the driving motors.

1. INTRODUCTION

Ball launchers are quite popular among tennis enthusiasts and there are many different types of such training machines available in the market. The main difference between them lies in the number of controlled parameters and possible ranges of their adjusting.

Generally, regarding the way the ball is thrown, tennis ball launchers can be divided into two groups: 1) the machines using compressed air – further called as pneumatic launchers, and 2) the machines using rotating rollers (or wheels) – further called as mechanical launchers.

In pneumatic launchers the ball is thrown using the air jet. The air is compressed with the attached compressor and stored inside the chamber. The initial velocity of the ball depends on the output pressure of the compressor. Elevation and heading angles can be adjusted by setting the direction of the outlet tube. The tube can be set manually or using electric motors. The spin can be added by using the special tube ending (adaptor) slowing down one side of the ball, what makes it rotate about the required axis. The adaptor needs to be rotated about the tube's axis in order to change between the slice and the top spin. However, the spin velocity cannot be set separately, as it depends on the initial velocity of the throw.

Due to the compressor, pneumatic launchers are powered from the mains voltage of 230/110 V. They offer long durability, reliability, and resistance to various weather conditions. However, pneumatic launchers allow one to practice only basic strokes and cannot be used for more complex training programs. That is why they are mainly chosen by beginners.

Mechanical launchers shoot the ball by pulling it between two counter rotating rollers. The initial velocity of the ball depends on the rotating velocity of the rollers. The heading angle can be changed by turning the whole machine, or only its launching mechanism, left/right (yawing the rollers). Similarly the elevation angle can be changed by turning the launching mechanism up/down (pitching the rollers). The spin can be added by changing the rotating velocity of one (or two) rollers. The given model of the launcher can only have the rollers positioned horizontally (or vertically) what restricts the possible planes of ball rotation.

Usually, mechanical launchers are powered from accumulators and that is why they can be used at the courts with no mains supply or in case of its malfunction. However, accumulators could also be the disadvantage, as they restrict the time of the uninterruptible operation. When compared with pneumatic mechanisms the rollers provide better accuracy and a wider range of the possible strokes. They allow one to intensify his/her efforts and to practice more advanced training programs.

Analyzing both types of tennis launchers it can be stated that better parameters and a greater control potential have mechanical launchers. The launching mechanism in the form of two rollers provides a great repeatability, increases the initial velocity and makes it possible to smoothly and accurately control the flying velocity of the ball. It can also spin the ball in a required manner. Mechanical launchers can control the throw better; in case of pneumatic machines the ball hit by the air jet rolls inside the outlet tube in an unpredictable manner. This introduces many problems making it difficult to control the throw and the flight of the ball and resulting in worse accuracy and repeatability of pneumatic machines.

To the authors' knowledge there are hardly no research articles concerning the problem of designing tennis ball launchers. Existing reports focus mainly on the aerodynamics of different flying sport balls (Alam, 2007, Goodwill, 2004, Mehta, 1985, Naumov, 1993, Sayers, 1999), especially on the problem of calculating the Magnus force (Alam, 2007, Goodwill, 2004), determining drag and lift coefficients (Alam, 2007) or on the problems of hitting or bouncing the ball in a required manner (Sayers, 1999). That is why we decided to create a simple mathematical model of the mechanical tennis launcher, to simulate its behavior, and establish the main principles of their designing.

The aim of the present work is to analyze the possibilities of improving the performance and training possibilities of mechanical launchers. By studying the mathematical basis of the throw and the flight of the ball the initial parameters for the required trajectory are determined. The parameters include: initial elevation and heading angles, rollers angle, rotational speeds and powers of the motors driving the rollers. These can be very helpful for the design of the new mechanical tennis ball launcher.

2. BRIEF FORDESIGN

For the purposes of the mathematical analysis, the following brief fordesign for the new launcher were formulated:

- the ball can be thrown from different altitudes in order to practice the return of serve; the maximum altitude of the throw can be set to 3 meters above the court level (average player's height + arm's reach + length of the racket ≈ 3 meters) with the step value of 0.1 meter;
- initial velocity of the balls can be set from 30 to 200 km/h with the step value of 1 km/h; the minimum velocity is for the youngest players; the maximum – is the average serve velocity of the advanced players with the several years' playing experience;
- throw angles:
  - elevation angle can be set from -20 to 80 degrees (from -0.38 to 1.4 radians) with the step value of 1 degree; the negative angles are needed to practice the return of serve; the maximum angle is to practice the lob strokes;
  - heading angle can be set from -40 to 40 degrees (from -0.7 to 0.7 radians) with the step value of 1 degree; the minimum/maximum angles are chosen in such a way that the ball can be passed from one sideline to the other near the net;
- time period between subsequent ball shots (launch rate) can be set from 1 to 15 seconds with the step value of 1 second; the minimum is the average value during the normal play; the maximum allows the coach to comment the last return of the player;
- the ball can be rotated around its axis in any plane by rotating the throwing mechanism around its symmetry axis from -180° to +180° (from -3.14 to 3.14 radians) with the step value of 1 degree;
- rotating speed of the ball can be set from 0 to 2000 rpm (from 0 to 209,4 rad/s) with the step value of 1 rpm; the maximum is the average value of the advanced players;
- the points at the court surface at which the ball should bounce can be freely chosen – the same points can be hit by different launch parameters depending on the practiced stroke.

3. MODEL OF THE FLYING BALL

The flying ball is impacted by three main forces: gravity, drag and lift (Fig. 1).

Only the drag force resulting from the translational motion of the ball is considered. The influence of the drag resulting from the rotation is neglected. The video recordings confirm that during the hit at the court the rotational speed of the ball is almost the same as just after the serve, i.e. the rotational speed drop during the flight is quite negligible. This means that the moment of the air drag force during the rotation does not influence the flight of the ball in a noticeable manner. However, the lift force resulting from the rotation of the ball around its axis; i.e. from the so called Magnus effect is taken into consideration.

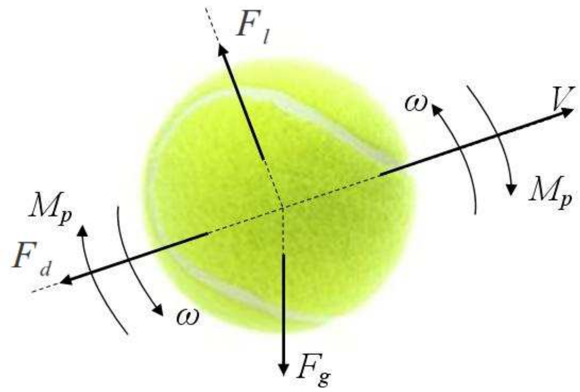

Fig. 1. Forces: gravity F_g, lift (Magnus) F_l, drag F_d, and drag moment M_p, and velocities: translational V and rotational ω of the flying ball

Drag force $\overline{F}_d$ is the component of the aerodynamic force appearing during the motion of the solid. It acts opposite to the direction of motion (Prosnak, 1970):

$$\overline{F}_d = -\tfrac{1}{2} c_d \rho \pi r^2 V \overline{V} \tag{1}$$

where: c_d is the drag force coefficient depending on the shape of the solid, ρ is air density, r is ball radius, and V – ball translational velocity.

Magnus effect lies in the generation of the lift force perpendicular to the translational velocity of the cylinder (or other solid of revolution, e.g. the ball) spinning in the surrounding fluid (Fig. 2). Rotating ball influences the surrounding air and makes it rotate too. On the other hand, the air pushing the ball in the translational motion flows AT one side of the ball at the same direction as the rotation of the ball. At this side the air is accelerated and its pressure drops. At the other side the flow direction of the air is opposite to the rotation. This decelerates the air and increases its pressure. Consequently, the pressure difference between the two sides of the ball evolves and changes the motion trajectory of the ball.

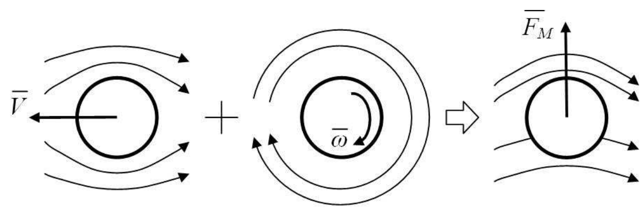

Fig. 2. Magnus effect: Magnus force F_M, translational V, and rotational ω velocities

Lift force $|\bar{F}_l|$ can be calculated, as (Prosnak, 1970):

$$\left|\bar{F}_l\right| = \tfrac{1}{2} c_l \pi r^2 \rho \bar{V} V \tag{2}$$

where c_l is the lift force coefficient.

Introducing rotational speed $\bar{\omega}$ instead of the scalar linear velocity V in Eq. (2), the Magnus force for the ball can be presented, as:

$$\bar{F}_l = \tfrac{1}{2} c_l \pi r^3 \rho \bar{V} \times \bar{\omega} \tag{3}$$

The values of the lift force and similarly of the drag force coefficients can be determined experimentally in a wind tunnel. Alam et al., 2007) investigated different tennis balls for various translational and rotational speeds. They found out that the drag coefficient changes from 0,55 to 0,85, while the lift coefficient – from 0,30 to 0,70 if translational speeds ranging from 20 to 140 km/h and rotational speeds from 0 to 3000 rpm are considered. By comparing the computational and experimental results they stated, that lift and drag forces depend not only on the speed of the ball, but also on the state (roughness) of its surface. Similar results obtained Goodwill et al. (Goodwill, 2004) who in a wind tunnel investigated aerodynamic properties of a range of new and used tennis balls for a velocity range from 20 to 60 m/s. Mehta (Mehta, 1985, 2004) presented many visualizations and obtained sets of drag and lift coefficient values for different sports balls spinning in a wind tunnel. Sayers and Hill (Sayers, 1999) presented experimental results of drag and lift coefficients for stationary and rotating cricket balls. Naumov et. al. (Naumov, 1993) compared numerical and experimental results of the research of the falling ball and determined the influence of the initial angular velocity on the deviation from the vertical.

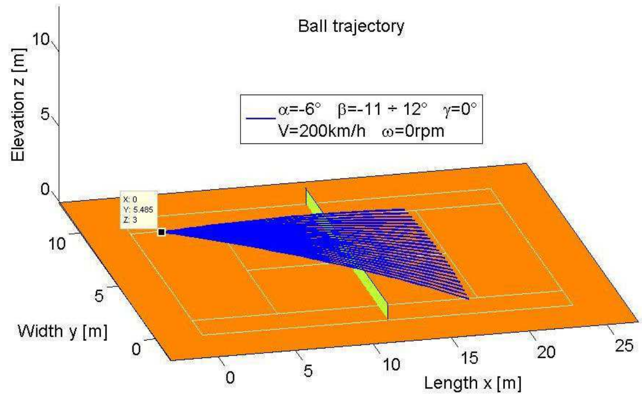

Fig. 3. Ball trajectories for different heading angles β: 3D view

Although aerodynamics problems of the flying sport balls, especially the Magnus effect, have been studied by many researches, there is no acceptable solution and explanation to this problem up till now. Thus, in the present study the simplified model will be applied, in which no additional effects such as 3D air flow around the ball or the flow turbulence will be included.

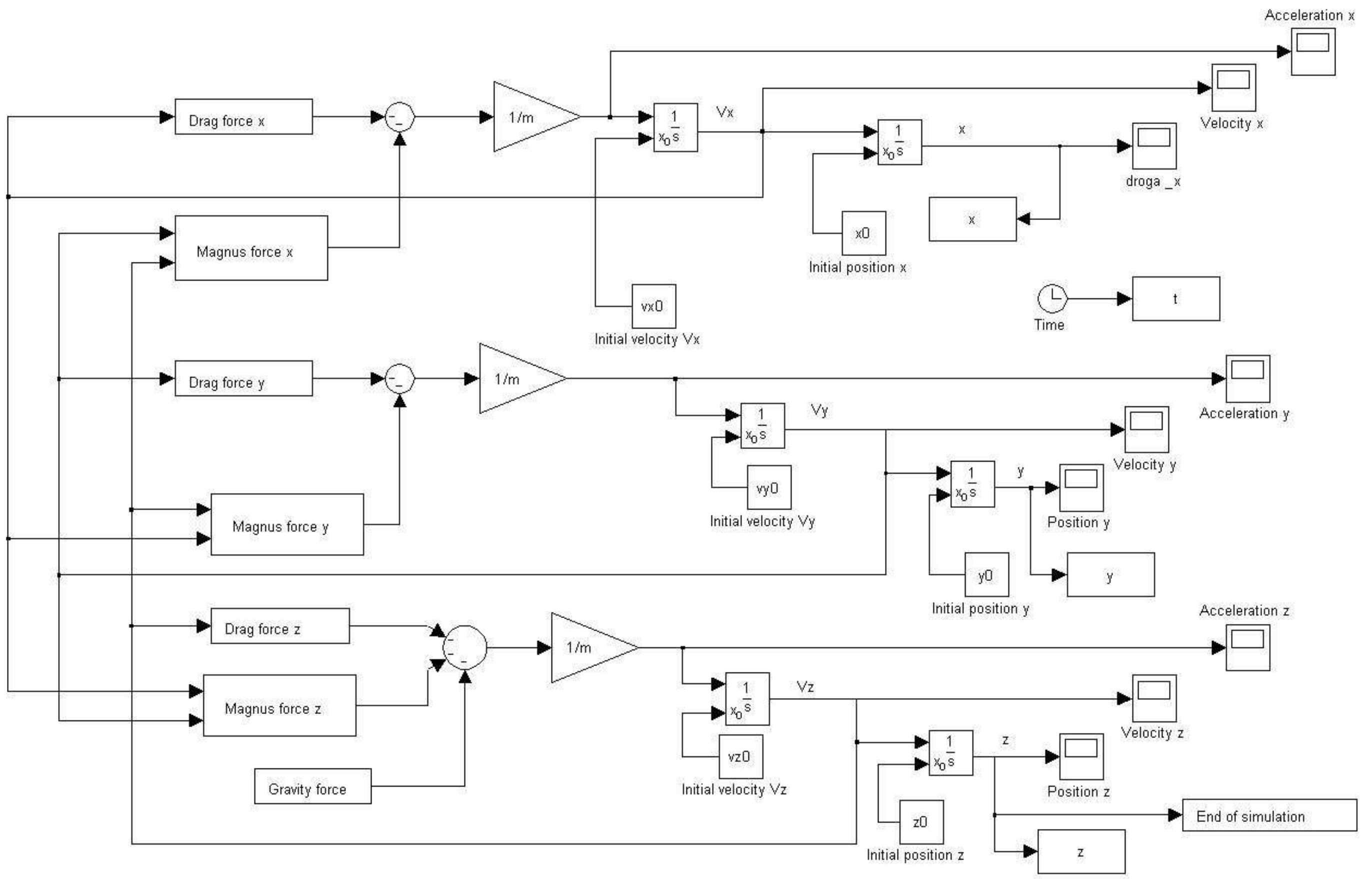

Fig. 4. Block diagram of the flying ball simulation model

The general motion equation of the flying ball can be presented, as:

$$m\frac{d^2\overline{x}}{dt^2} = \overline{F}_g + \overline{F}_d + \overline{F}_l \tag{4}$$

Applying Eqs. (1), (3), after projecting the above equation on the three axes of the Cartesian coordinate system, the following formulas for x, y, and z directions are obtained

$$m\frac{d^2x}{dt^2} = -\tfrac{1}{2}r^2c_d\pi\rho V_x\frac{dx}{dt} - \tfrac{1}{2}c_l r^3\pi\rho\left(\frac{dy}{dt}\omega_z - \frac{dz}{dt}\omega_y\right) \tag{5}$$

$$m\frac{d^2y}{dt^2} = -\tfrac{1}{2}r^2c_d\pi\rho V_y\frac{dy}{dt} - \tfrac{1}{2}c_l r^3\pi\rho\left(\frac{dz}{dt}\omega_x - \frac{dx}{dt}\omega_z\right) \tag{6}$$

$$m\frac{d^2z}{dt^2} = -mg - \tfrac{1}{2}r^2c_d\pi\rho V_z\frac{dz}{dt} - \tfrac{1}{2}c_l r^3\pi\rho\left(\frac{dx}{dt}\omega_y - \frac{dy}{dt}\omega_x\right) \tag{7}$$

where V_x, V_y, V_z and ω_x, ω_y, ω_z are the components of the translational V and rotational ω speeds, m is the mass of the ball, g is gravity acceleration.

Based on Eqs. (5)-(7), the simulation model of the flying ball was created in Matlab/Simulink. The model allows one to simulate various trajectories of the flying ball in dependency of initial throw parameters: elevation α, heading β, rotation γ angles, as well as, linear V, and rotational ω speeds of the ball.

The block diagram of the simulation model is presented in Fig. 4. The drag and lift coefficients can be introduced directly by the user or calculated from a specially designed function approximating the values given by Alam et al., (2007). The user decides which values are chosen for further calculations. This way, he/she can easily analyze what is the influence of the drag or lift force on the ball trajectory. However, he/she must be aware, that the reliable results can be obtained only, if the values suggested by Alam et al., (2007) or other researches are chosen, and that the allowable translational (20 to 140 km/h) and rotational (0 to 3000 rpm) initial speeds are not exceeded.

Several shots, e.g. serves differing only in the heading angle β can be presented at the same diagram, as it is shown in Fig. 3. The trajectory can be plotted in one of three planes, xy, xz, or yz. The simulations in Fig. 3 were conducted for the typical tennis ball of mass $m = 0{,}058$ kg, and radius $r = 0{,}333$ m.

4. ANALYSIS OF THE BALL THROW

In order to throw the ball using two counter-rotating rollers two conditions should be met. The first is the condition for pulling the ball in between the rollers. The second is the condition for throwing the ball away from between the rollers.

4.1. Pulling the ball

When the ball is being pulled by the rollers, two main forces appear at the contact points between the ball and the rollers. These are: the pressure force N, which is normal to the contact surface, and the tangent friction force T (Fig. 5). The ball can be pulled in between the balls only if the horizontal component T_h, of the friction force T is equal or greater than the horizontal component N_h of the pressure force N (Wusatowski, 1960).

$$T\cos\lambda \geq N\sin\lambda \tag{8}$$

where λ is the so called “grab angle” between the plane containing the axes of the rollers and the plane containing the axis of one of the rollers and the contact point between the ball and the roller.

As $T = \mu N$, where μ is the friction coefficient, then:

$$\mu \geq tg\lambda \tag{9}$$

Having in mind that:

$$\mu = tg\eta \tag{10}$$

where η is friction angle, the condition for dragging the ball in between the rollers can be presented, as:

$$\lambda \leq \eta \tag{11}$$

Friction coefficient μ will be determined experimentally, what is presented further.

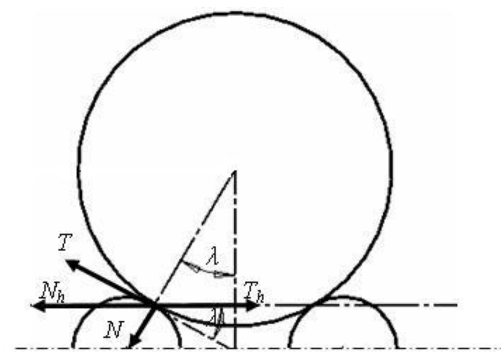

Fig. 5. Forces and geometry at the contact between the ball and the rollers during pulling the ball in between the rollers: horizontal component N_h of the pressure force N, horizontal component T_h of the friction force T, grab angle λ

4.2. Throwing the ball

The condition for throwing the ball away from between the rollers states that the friction force between the ball and the rollers should be equal or greater than the inertia force of the ball (Fig. 6).

This condition ensures that there is no slip between the rollers and the ball:

$$F_w \geq 2T \tag{12}$$

The inertia force depends on the required linear velocity of the flying ball, or more exactly on its acceleration that should be transmitted from the rollers in order to obtain the required initial velocity of the throw.

The friction force between the rollers and the ball depends on the friction coefficient and on the pressure force between the rollers and the ball. The pressure force can be obtained from the definition of Young’s modulus for the ball:

$$N = \frac{Es\Delta l}{2r} \tag{13}$$

with Young's modulus E, ball deformation Δl and the area at the deformation surface s.

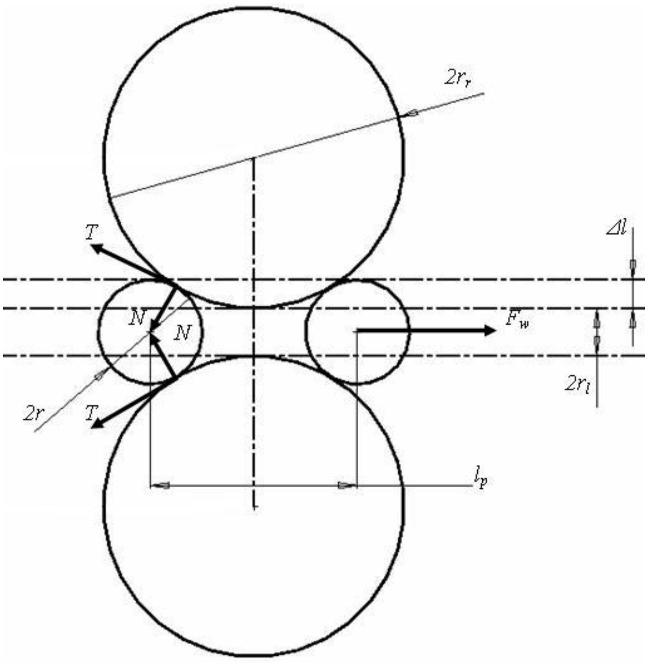

Fig. 6. Forces and geometry at the contact between the ball and the rollers during throwing the ball away: ball inertia force F_w, pressure force N, friction force T, ball radius r, rollers radius r_r, distance between rollers l_p, radius r_l of the ball deformed (squeezed) by the rollers, ball deformation Δl due to the rollers pressure

For further analysis following parameters were assumed: rollers radius $r_r = 0{,}1$ m, and distance between axes of the rollers $d_{sr} = 0{,}25$ m. The Young's modulus will be determined experimentally, what is presented further.

5. EXPERIMENTAL INVESTIGATION OF BALL PARAMETERS

The values of the Young's modulus for the ball and the friction coefficient between the ball and the plastic coverings of the rollers were determined experimentally.

5.1. Young's modulus

Young's modulus was determined by measuring the value of the ball deformation resulting from the applied load force. The ball was located between two flat plates (Fig. 7).

The upper plate was gradually loaded with ten weights of 0,55 kg each, i.e. the overall loading weight changed from 0,55 kg to 5,5 kg. The deformation was measured along three different axes of the ball. As the final result for the given load the arithmetic average of the results for each of the axes was assumed. The values obtained for three different axes of the ball differed in about 3%. These differences were due to the irregular structure of the ball, e.g. at the location of the seams, the ball did not deform evenly.

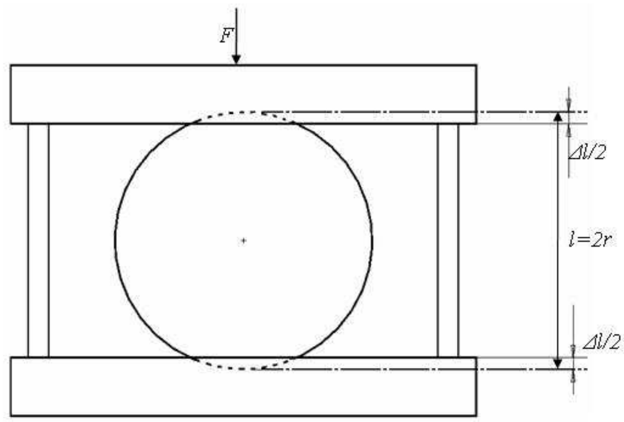

Fig. 7. Schematic diagram of the test stand for Young's modulus measurements

Several tests of the examined ball were performed. Having the loads and corresponding deformations Young's moduli were calculated. Their values are presented in Tab. 1. The dependency between the loading force and the deformation of the examined ball is shown in Fig. 8, where the Young's modulus can be read out as the tangent of the curve's slope angle.

The averaged value of the Young's modulus for the examined ball is 3,63 MPa. The smallest value of 2,83 MPa was obtained for the loading weight of 1,65 kg, while the biggest (4,50 MPa) – for the loading weight of 4,95 kg. The variability range is 1,60 MPa for all loads.

Tab. 1. Results of the Young's modulus measurements for the examined tennis ball

Loading weight [kg]	Loading force [N]	Averaged ball deformation [m]	Calculated Young's modulus [MPa]
0,55	5,40	0,0005	3,40
1,10	10,79	0,0011	3,09
1,65	16,19	0,0018	2,83
2,20	21,58	0,0021	3,24
2,75	26,99	0,0025	3,40
3,30	32,37	0,0029	3,52
3,85	37,77	0,0030	3,96
4,40	43,16	0,0033	4,12
4,95	48,56	0,0036	4,50
5,50	53,96	0,0040	4,25

In order to assess the correctness of the experimental tests, the results obtained for the examined ball were compared with the results calculated for the so called "standardized" tennis balls.

The requirements concerning loads and deformations for the standardized tennis balls are strictly defined (Romer, 2005). Three different types of the standardized balls are allowed to be used for playing tennis. Primary deformation for the type 1 (fast) ball should be greater than 0,495 cm (0,19 in) and less than 0,597 cm (0,24 in), while secondary deformation should be greater than 0,637 cm (0,26 in) and less than 0,914 cm (0,36 in) if the loading weight of 8,165 kg (18 lb) is applied. For type 2 (medium) and type 3 (slow) balls primary deformation should be greater than 0,559 cm (0,22 in) and less than 0,737 cm (0,29 in), while secondary deformation should be greater than 0,800 cm (0,31 in) and less than 1,080 cm (0,43 in) for the same loading weight of 8,165 kg. The primary deformation is measured directly for the given load of 8,165 kg, while the secondary one is also for the load of 8,165 kg but just after applying the load which reduces the ball diameter to 2,54 cm (1 in). These two deformations should be the averaged values for three different axes of the ball, while two subsequent measurements cannot differ more than for 0,076 cm (0,03 in).

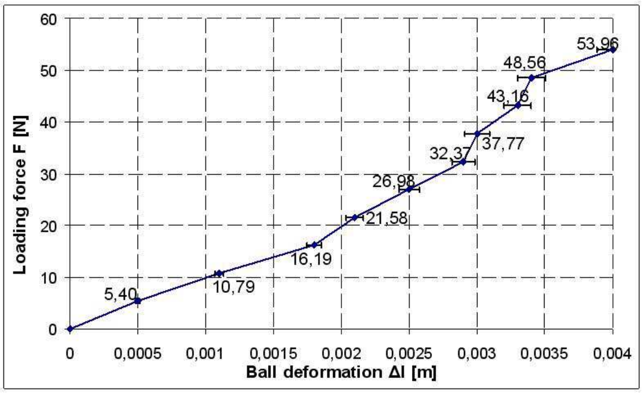

Fig. 8. Force versus ball deformation

The Young's moduli for the standardized tennis balls were calculated for two deformation limits of each type. They are presented in Tab. 2. The averaged value of the Young's modulus for the standardized ball is 4,20 MPa, with the variability range of 1,68 MPa.

As can be seen the values of the Young's modulus for the examined ball and for the standardized balls are similar. However, for further calculations the value of 4,20 MPa was assumed.

Tab. 2. Results of the Young's modulus calculations for the standardized tennis balls

Loading weight [kg]	Loading force [N]	Averaged ball deformation [m]	Type of the ball	Calculated Young's modulus [MPa]
8,165	80,1	0,00495	Fast	5,10
8,165	80,1	0,00597	Fast	4,22
8,165	80,1	0,00559	Medium	4,51
8,165	80,1	0,00737	Medium	3,42
8,165	80,1	0,00559	Slow	4,51
8,165	80,1	0,00737	Slow	3,42

5.2. Friction coefficient

Friction coefficient between two bodies in contact can be experimentally determined by locating the tested body on an inclined plane and gradually changing its slope angle (Fig. 9). The angle η at which the tested body begins to slide down is the so called friction angle, from which the friction coefficient μ can be calculated (see Eq. (10)).

In order to determine the friction coefficient between the real tennis ball and the roller, this simple test stand was modified by placing the ball not on the plane, but on the roller segment located on the inclined plane (Fig. 11). Thus, following configurations were tested

- the ball on the inclined plane (Fig. 10);
- the ball on the roller segment located on the inclined plane (Fig. 11).

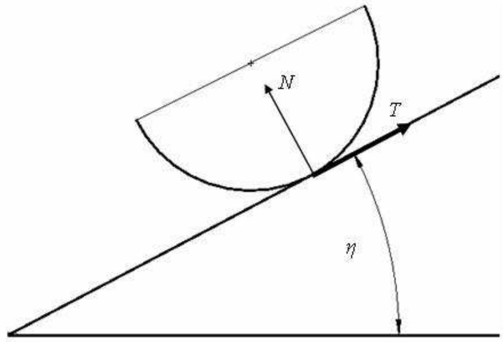

Fig. 9. Derivation of friction coefficient on an inclined plane: friction force T, pressure force N, friction angle η

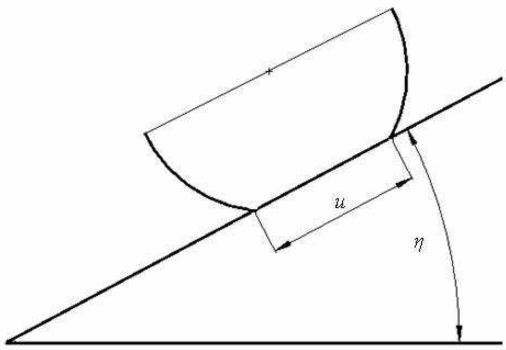

Fig. 10. Derivation of friction coefficient between the ball and the inclined plane: friction angle η, length in contact u

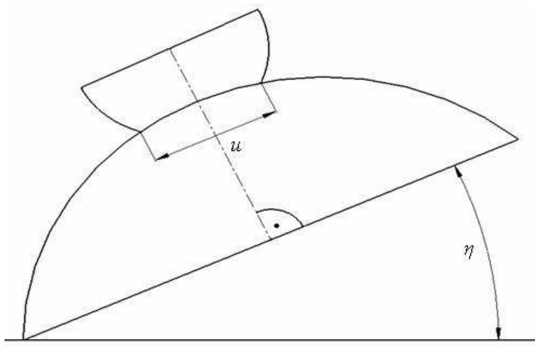

Fig. 11. Derivation of friction coefficient between the ball and the roller segment: friction angle η, length in contact u

The normal tennis ball was cut into two equal pieces along its diameter. In the first case the cut half was located on the inclined plane, and in the second – on the roller segment located on the inclined plane. The plane and the roller segment were made of plastic (polypropylene). The inner part of the half of the ball was pressed in order to increase the contact surface between the ball and the roller, i.e. to better reproduce the real throw. Next, the slope angle at which the ball started to slide down, and the length of the ball in contact with the plane (or with the roller) were measured. The results are presented in Tab. 3.

Tab. 3. Experimental determination of friction coefficient – measurements and calculations

Configuration	Length in contact [m]	Friction angle [°]	Friction coefficient
Inclined plane	0,008	32	0,63
	0,010	34	0,67
	0,015	37	0,75
	0,020	39	0,80
	0,025	41	0,87
	0,030	43	0,93
	0,035	44	0,97
Roller on an inclined plane	0,005	30	0,58
	0,010	34	0,67
	0,015	40	0,84
	0,020	46	1,04
	0,025	53	1,33
	0,030	59	1,66
	0,035	64	2,05

Figures 12 and 13 present the friction coefficient versus the length in contact for two considered cases. As can be seen the friction coefficient increases for higher contact lengths. Its value depends also on the shape of the two bodies in contact – for the given contact length the friction coefficient is higher for the roller segment than for the plane.

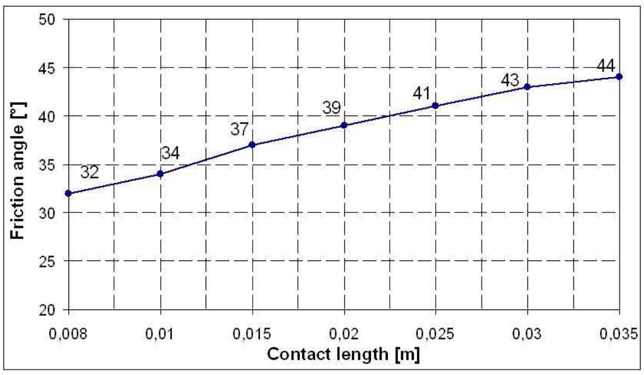

Fig. 12. Friction coefficient versus length in contact; the ball on the inclined plane

During the throw the ball is squeezed by the rollers, the contact length (surface) increases, and the pressure forces act in different directions (normally to the contact surface) what results in the increase of the friction force. That is why the value of 2,05 for friction coefficient was chosen for further calculations. This is the value obtained for the maximum pressure and maximum contact length between the ball and the roller segment. It is supposed, that such configuration resembles the real ball between the rollers best.

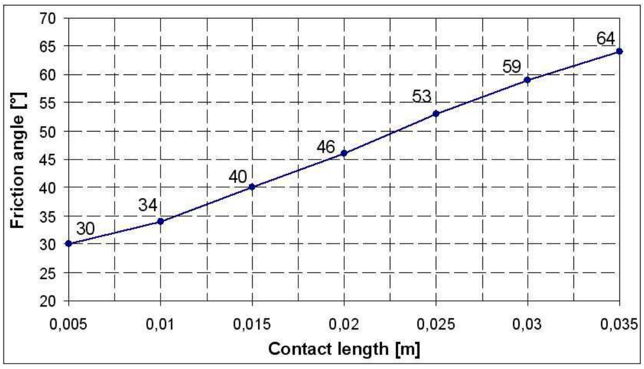

Fig. 13. Friction coefficient versus length in contact; the ball on the roller segment

6. ROTATING SPEEDS OF THE ROLLERS

The linear and angular speed of the ball depend the circumferential speeds of the rollers:

$$V = \frac{V_1 + V_2}{2} \tag{14}$$

$$\omega r_l = \frac{V_2 - V_1}{2} \tag{15}$$

where V_1, V_2 – circumferential speeds of the first and the second roller, r_l – ball radius after deformation (Fig. 14).

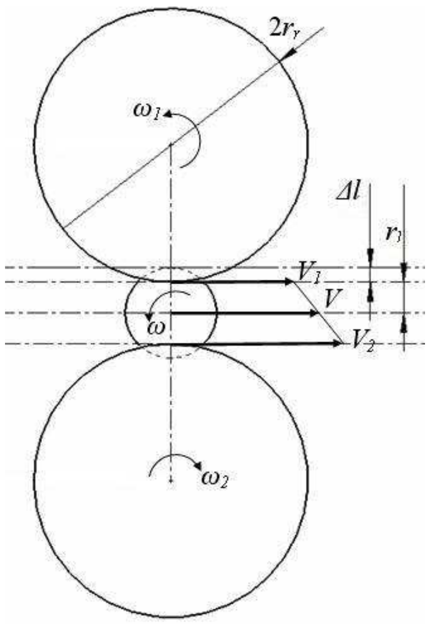

Fig. 14. Velocities during the throw: V, ω – linear and rotational speeds of the ball, V_1, V_2, ω_1, ω_2 – circumferential and rotational speeds of the rollers, r_r – rollers radius, r_l – radius of the deformed ball, Δl – ball deformation due to the rollers pressure

If the ball has to rotate about its own axis after leaving the launcher, the circumferential speeds of the rollers should be different. Otherwise, the ball will have no rotation.

Given the required linear and rotational speeds of the flying ball, the rotational speeds of the rollers can be calculated, as:

$$\omega_1 = \frac{V - \omega r_l}{r_r} \tag{16}$$

$$\omega_2 = \frac{2V}{r_r} - \omega_1 \tag{17}$$

From Eqs. (16) and (17) it is clear, that for the maximum linear speed of 200 km/h with no ball rotation, the rotational speed of the rollers should be 5300 rpm. Similarly, for the ball flying with its maximum linear (200 km/h) and rotational (2000 rpm) speeds, the rollers should rotate with velocities $\omega_1 = 5800$ rpm and $\omega_2 = 4800$ rpm.

7. POWER OF THE MOTORS

During the throw, the ball takes some energy from the rollers while going through them. This decreases the rotational speeds of the rollers.

Kinetic energy change of the rollers during the throw can be presented, as the difference between their final E_k^r and initial E_{k0}^r energies:

$$\Delta E_k^r = E_k^r - E_{k0}^r = \tfrac{1}{2} I_r \left[\left(\omega_1^2 + \omega_2^2 \right) - \left(\omega_{10}^2 + \omega_{20}^2 \right) \right] \tag{18}$$

with roller moment of inertia I_r,

$$I_r = \tfrac{1}{2} m_r r_r^2 \tag{19}$$

and rotational speeds of the rollers: initial ω_{10}, ω_{20}, and during the throw, and ω_2.

For the ball it can be assumed, that its initial kinetic energy is zero, while the final energy (just after the throw) depends on its linear and rotational speeds:

$$\Delta E_k^p = {}^k E_k^p = \tfrac{1}{2} m V^2 + \tfrac{1}{2} I_p \omega^2 \tag{20}$$

where the ball's moment of inertia I_p is given by:

$$I_p = \tfrac{2}{3} m r^2 \tag{21}$$

Including Eqs. (17) and (19) the required initial kinetic energy of the ball can be calculated, as:

$$E_{k0}^r = \tfrac{1}{2} I_r \left({}^k\omega_1^2 + {}^k\omega_2^2 \right) + \left(\tfrac{1}{2} m V^2 + \tfrac{1}{2} I_p \omega^2 \right) \tag{22}$$

Let us assume, that:

$$\frac{\omega_{10}}{\omega_{20}} \cong \frac{\omega_1}{\omega_2} \tag{23}$$

Then, the initial rotational speeds of the rollers, needed to throw the ball with the required linear V and rotational ω velocities can be calculated, as:

$$\omega_{10} = \omega_1 + \sqrt{\tfrac{1}{2} I_p \omega^2} \tag{24}$$

$$\omega_{20} = \omega_2 + \sqrt{\tfrac{1}{2} I_p \omega^2} \tag{25}$$

As we can see the rotational speeds of the rollers should be increased by a small value depending on the required rotational speed of the ball. For the ball flying with its maximum rotational speed of 2000 rpm, the rotational speeds of the rollers should be increased by 90 rpm (i.e. by 1,5% for the first, and by 1,8% for the second roller).

The power of the motors driving the rollers can be determined by calculating separately the work done by the motors during the start-up and the work needed to throw the ball.

Including Eqs. (17) and (21) the work needed to throw the ball can be expressed, as:

$$\begin{aligned} \Delta W_d^r = E_k^r - E_{k0}^r = \left[\tfrac{1}{2} I_r \left({}^k\omega_1^2 + {}^k\omega_1^2 \right) \right] - \\ \left[\tfrac{1}{2} I_r \left({}^k\omega_1^2 + {}^k\omega_1^2 \right) + \tfrac{1}{4} \left(m V^2 + I_p \omega^2 \right) \right] = \\ - \tfrac{1}{4} \left(m V^2 + I_p \omega^2 \right) \end{aligned} \tag{26}$$

where the minus sign denotes the fact, that the energy was returned by the rollers.

On the other hand, the work needed to start-up the rollers can be presented, as:

$$\Delta W_r^r = E_{k0}^r = \tfrac{1}{2} I_r \left(\omega_1^2 + \omega_1^2 \right) + \tfrac{1}{4} \left(m V^2 + I_p \omega^2 \right) \tag{27}$$

that is for the first roller:

$$\Delta W_r^{r1} = \tfrac{1}{2} I_r \omega_1^2 + \tfrac{1}{4} \left(m V^2 + I_p \omega^2 \right) \tag{28}$$

and for the second:

$$\Delta W_r^{r2} = \tfrac{1}{2} I_r \omega_2^2 + \tfrac{1}{4} \left(m V^2 + I_p \omega^2 \right) \tag{29}$$

The required power depends on the throw time t_d or on the time of starting-up t_r:

$$P_d = \frac{W_d}{t_d} \tag{30}$$

$$P_r = \frac{W_r}{t_r} \tag{31}$$

For the minimum throw time of $t_d = 1$ s (see the introductory requirements), the power needed to drive the rollers and throw the ball is $P_d = 45$ W. On the other hand for the 45 W motors, the time needed to start-up the rollers to their nominal rotational speeds is $t_r = 35$ s, what is the acceptable value.

8. CONCLUSION

Based on the mathematical analysis several conditions and parameters for the design of the new type of the tennis ball launcher have been found.

The parameters of the typical ball such, as its Young's modulus and friction coefficient have been experimentally tested and determined. Using the mathematical model of the ball between two counter rotating rollers, these allowed us to calculate the required initial parameters of the throw, i.e.

the rotational speeds of the rollers and the power of the motors.

The presented model of the flying tennis ball is quite simple, yet it allows one to analyze the trajectories of the ball for different initial conditions in a reliable manner. The results of such analysis can be very useful while implementing the control system for the new tennis ball launcher. Based on calculated ball trajectories, the database of different possible hits and training programs can be created and implemented in the control system.

Using the presented analysis and its results the authors are currently working on the design and technical implementation of the new type of the tennis ball launcher.

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ANALIZA MATEMATYCZNA NOWEGO TYPU WYRZUTNI PIŁEK TENISOWYCH

Streszczenie: W artykule przeprowadzono analizę matematyczną nowego typu wyrzutni piłek tenisowych, na podstawie której oceniono możliwości technicznej implementacji takiego urządzenia. Opisano tradycyjne rodzaje wyrzutni, a następnie sformułowano wymagania usprawniające możliwości treningowe takich maszyn. Sformułowano i numerycznie rozwiązano równania lotu piłki tenisowej, umożliwiające analizę trajektorii przy danych warunkach początkowych: kątach wzniosu i szerokości oraz kącie obrotu rolek. Następnie przedstawiono wyniki analizy wyrzutu piłki wyrzucanej za pomocą dwóch przeciwnie wirujących rolek. Na podstawie przeprowadzonych badań doświadczalnych wyznaczono sztywność (moduł Young'a) oraz współczynniki tarcia typowej piłki tenisowej. Dzięki temu obliczono parametry początkowe wyrzutu: prędkości obrotowe rolek i moce silników napędowych.