MODELLING OF ADHESION EFFECT IN FRICTION OF VISCOELASTIC MATERIALS

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Abstract: A model is suggested for the analysis of the combined effect of viscoelastic properties of bodies and adhesive interaction between their surfaces in sliding. The model is based on the solution of the contact problem for a 3D wavy surface sliding on the boundary of a viscoelastic foundation taking into account the molecular attraction in the gap between the bodies. The influence of adhesion on the contact stress distribution, real contact area and hysteretic friction force is analyzed.

1. INTRODUCTION

According to the molecular-mechanical theory of friction (Kragelski, 1949), the friction force consists of two components. The deformation component arises due to deformation of materials by surface asperities. The adhesion component is due to molecular forces between surfaces.

The deformation component of the friction force of viscoelastic bodies can be determined by calculating the hysteretic losses as a result of cyclic deformation of surface layers by asperities of rough surfaces during their mutual sliding (Goryacheva, 1998).

Molecular forces appear in the gap between surfaces and act at a distance specified by the potential of molecular interaction (Deryagin et al., 1985). Molecular interaction between the surfaces leads not only to tangential traction giving the adhesion component of the friction force. At micro- and nano-scale levels of the surface roughness, at which the value of the gap is comparable with the radius of adhesive forces action, molecular forces acting in normal direction to contact surface can also influence the deformation component of the friction force. The influence of the adhesion attraction between surfaces on the hysteretic friction force was analyzed (Makhovskaya, 2005) for a separate asperity with a given shape of tip.

In what follows, the effect of adhesion on the friction In sliding of rough viscoelastic surfaces is modeled with taking into account the whole surface geometry - both tips of asperities and valleys between them for a 3D rough surface. Previously a similar approach was used for a 2D rough surface (Goryacheva and Makhovskaya, 2010).

2. DESCRIPTION OF THE MODEL

Consider a rigid wavy surface sliding with the velocity V along the x-axis on the viscoelastic foundation. The shape of the wavy surface is described by the periodic function:

$$f(x,y) = h - \frac{h}{4}\left(\cos\left(\frac{2\pi x}{l}\right)+1\right)\left(\cos\left(\frac{2\pi y}{l}\right)+1\right), \tag{1}$$

where h and l are the height of asperities and distance between them, respectively, $h \ll l$ (Fig. 1).

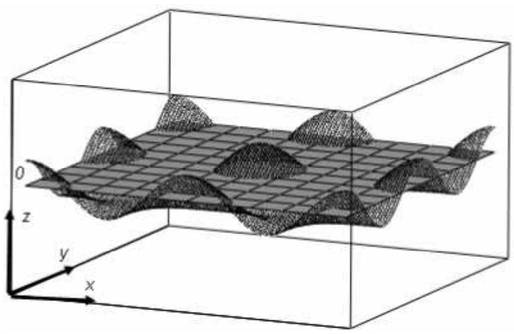

Fig. 1. Scheme of contact between a rigid wavy surface and a viscoelastic foundation

The mechanical properties of the viscoelastic foundation are described by the linear 1-D model:

$$w + T_\varepsilon \frac{\partial w}{\partial t} = \frac{(1-\nu^2)H}{E}\left(p + T_\sigma \frac{\partial p}{\partial t}\right) \tag{2}$$

where p and w are the pressure and displacement on the boundary of the viscoelastic foundation, E is the Young modulus, ν is Poisson's ratio, H is the thickness of the viscoelastic layer, T_ε and T_σ are the retardation and relaxation times, respectively. Since $h \ll l$, we assume that $\partial w_z/\partial x \ll 1$ and then the quantities p and w are approximately equal to their projections on the z-axis, p_z and w_z, respectively.

Let the system of coordinates (x', y', z') be connected with the viscoelastic foundation, and the system of coordinates (x, y, z) with the sliding wavy surface so that:

$$x' = x + Vt, \quad y' = y, \quad z' = z \tag{3}$$

In the moving system of coordinates *(x, y, z)* relation (2) has the form:

$$w - VT_\varepsilon \frac{\partial w}{\partial x} = \frac{(1-\nu^2)H}{E}\left(p - T_\sigma V \frac{\partial p}{\partial x}\right) \tag{4}$$

To take into account the adhesive (molecular) attraction between the surfaces, introduce the negative adhesive stress $p = -p_a(\delta)$ acting on the boundary of the viscoelastic foundation, where δ is the value of gap between the surfaces. We use the Maugis-Dugdale model in which the dependence of the adhesive stress on the gap between the surfaces has a form of one-step function (Maugis, 1991):

$$p_a(\delta) = \begin{cases} p_0, & 0 < \delta \le \delta_0 \\ 0, & \delta > \delta_0 \end{cases} \tag{5}$$

where δ_0 is the maximum value of gap for which the adhesive attraction acts. The surface energy γ is specified by the relation:

$$\gamma = \int_0^{+\infty} p_a(\delta)\, d\delta = p_0 \delta_0 \tag{6}$$

Since the wavy surface is periodic with the period l, the contact problem can be considered in a square region $x \in (-l/2; l/2); y \in (-l/2; l/2)$. This square contains one asperity of the periodic wavy surface. The conditions of periodicity $p(x,y) = p(x+l, y)$ and $w(x,y) = w(x + l, y)$ must be satisfied. In the moving system of coordinates (*x*, *y*, *z*), the following boundary conditions for the stresses and displacements take place at the foundation surface $(z = 0)$ in the square region region $x \in (-l/2; l/2); y \in (-l/2; l/2)$:

$$\begin{aligned} & w(x,y) = f(x,y) + D, && (x,y) \in \Omega^c; \\ & p(x,y) = -p_0, && (x,y) \in \Omega^a; \\ & p(x,y) = 0, && (x,y) \notin \Omega^c \cup \Omega^a; \end{aligned} \tag{7}$$

Here Ω^c is the contact region, Ω^a is the region in which adhesive stress $-p_0$ acts, and D is the penetration of the asperity into the foundation. The equilibrium condition is also satisfied:

$$P = \iint\limits_{\Omega^c \cup \Omega^a} p(x,y)\, dxdy \tag{8}$$

where P is the normal load applied to each asperity of the wavy surface.

3. METHOD OF SOLUTION

The contact problem is solved by using the strip method (Kalker, 1990) which is an exact method for the case of 1D foundation. The square region $x \in (-l/2; l/2)$; $y \in (-l/2; l/2)$ is divided into $2N$ strips of equal thickness Δ (Fig. 2). The normal displacement of the center of a strip *j* is:

$$D_j = \frac{h}{2}\left(\cos\left(\frac{2\pi\Delta j}{l}\right) - 1\right) + D \tag{9}$$

If the maximum normal penetration D of the wavy surface is prescribed, then for each strip, the maximum normal penetration is given by (9). The shape of rigid wavy surface in this strip is specified by:

$$f_j(x) = \frac{h_j}{2}\left(\cos\left(\frac{2\pi x}{l}\right) - 1\right), \quad h_j = \frac{h}{2}\left(\cos\left(\frac{2\pi\Delta j}{l}\right) + 1\right) \tag{10}$$

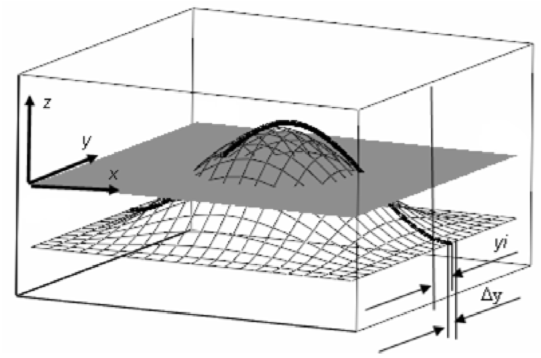

Fig. 2. The square $x \in (-l/2; l/2); y \in (-l/2; l/2)$ and its division into strips

In each strip, the contact problem is formulated and solved independently. For a *j*-th strip, the conditions for the displacement $w_j(x)$ and pressure $p_j(x)$ follow from (7) and they have the form:

$$\begin{aligned} & w_j(x) = f_j(x) + D_j, && x \in \Omega^c{}_j \\ & p_j(x) = -p_0, && x \in \Omega^a{}_j \\ & p_j(x) = 0, && x \notin \Omega^c{}_j \cup \Omega^a{}_j; \end{aligned} \tag{11}$$

When the contact problem is solved and the contact pressure $p_j(x)$ is calculated for each strip, the normal load acting on an asperity is calculated by the summation, which follows from (8):

$$P = 2\sum_{j=1}^{N} P_j, \quad P_j = \Delta \int_{-l/2}^{l/2} p_j(x)dx \tag{12}$$

If the maximum displacement D is unknown, while the load P is prescribed, then some initial values of D is set and the iteration procedure is applied to attain the prescribed value of the load P.

To determine the tangential stress applied to the rigid wavy surface from the viscoelastic foundation, we use the relation:

$$\tau_j(x) = p_j(x)\sin\left[\mathrm{arctg}\left(f_j{}'(x)\right)\right] \approx p_j(x) f_j{}'(x) \tag{13}$$

Then the tangential (friction) force acting on the asperity is calculated as:

$$T = 2\sum_{j=1}^{N} T_j, \quad T_j = \Delta \int_{l/2}^{l/2} \tau_j(x)dx \tag{14}$$

This force is different from zero because the pressure distribution is nonsymmetrical with respect to the axis of symmetry of the asperity due to hysteretic losses in the viscoelastic material. This force is called the hysteretic or deformation component of the friction force. The corresponding friction coefficient is determined from the relation:

$$\mu = T / P \tag{15}$$

Thus, the problem is reduced to solving a 2-D contact problem for each strip to determine the contact pressure distribution $p_j(x)$ in the contact region $x \in \Omega_j$ and the boundary of the region of adhesive interaction Ω^a, after which the friction force can be calculated in accordance with Eqs. (12)-(15).

In order to write the boundary conditions for the contact pressures and displacements in a strip, we should take into account various regimes of the gap filling.

4. PROBLEM SOLUTION IN A STRIP FOR DIFFERENT REGIMES OF GAP FILLING

Three possible regimes of the gap filling are considered: saturated contact (Fig. 3a), discrete contact with saturated adhesive interaction (Fig. 3b), discrete contact with zones of adhesive interaction and zones of free boundary (Fig. 3c). In each *j*-th strip, one of these regimes is realized, depending on the displacement of the center D_j of this strip and values of the problem parameters (mechanical and geometric characteristics of the interacting bodies, load, and sliding velocity).

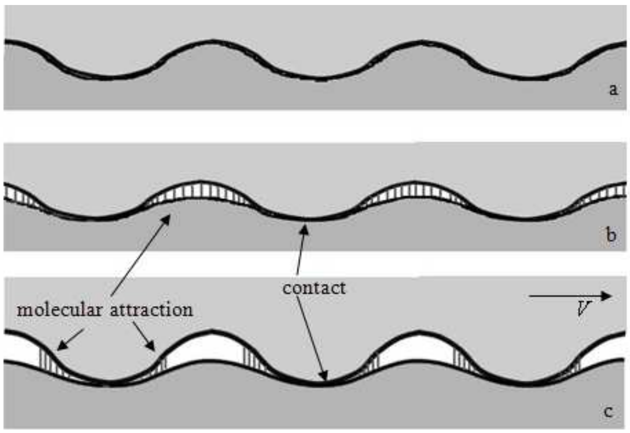

Fig. 3. Regimes og gap filling between the surfaces in the presence of adhesion

4.1. Saturated contact

In this case (Fig. 3,a), the displacement of the boundary $z = 0$ of the viscoelastic foundation $w_j(x)$ satisfies the first condition of (11) on the entire surface, i.e. over the whole length of the period $x \in (-l/2; l/2)$. By solving the differential equation (4) with the first condition of (11) and using the periodicity condition for the pressure $p_j(x - l/2) = p_j(x + l/2)$, we obtain the contact pressure in the form:

$$p_j(x) = \frac{E}{2H(l^2 + 4\pi^2 T_\sigma^2 V^2)} \Big[h_j (l^2 + 4\pi^2 T_\varepsilon T_\sigma V^2) \cos\frac{2\pi x}{l} + \\ + 2\pi l h_j V (T_\varepsilon - T_\sigma) \sin\frac{2\pi x}{l} + (4\pi^2 T_\sigma^2 V^2 + l^2)\left(2D_j - h_j\right) \Big] \tag{16}$$

where D_j and H_j are specified by relations (9) and (10). Normal force P_j acting on the j-th strip is determined by the relation:

$$P_j = \Delta \int_{-l/2}^{l/2} p(x) dx = \Delta \frac{El}{2Hl} (2D_j - h_j) \tag{17}$$

The saturated contact in the *j*-th strip is realized under the condition:

$$\min\left(p_j(x) \right) \geq -p_0 \tag{18}$$

Note that due to adhesion, the contact pressure can be negative but not smaller than the adhesive stress $-p_0$. If the minimum contact pressure in a *j*-th strip does not satisfy condition (18), then the saturated contact is not realized. In this case, the solution is sought for the discrete contact with saturated adhesive interaction.

4.2. Discrete contact with saturated adhesive interaction

In this case (Fig. 3,b), the problem solution is considered in the interval $x \in [-a_j, l - a_j]$. Two different boundary conditions take place for two zones of interaction. The differential equation (4) is solved in the zone of contact $-a_j < x < b_j$ for the contact pressure $p_j(x)$, the displacement $w_j(x)$ being specified by the first condition of (11). In the zone of adhesive interaction $b_j < x < l - a_j$, the differential equation (4) is solved for $w_j(x)$, the pressure $p_j(x)$ being prescribed by the second condition of (11). Thus two boundary conditions are necessary for the solution of these two differential equations. Also, two conditions are needed for the determination of the end points a_j and b_j of the contact region. As such conditions, the conditions of continuity of the functions $p_j(x)$ and $w_j(x)$ at the points $x = -a_j$ and $x = b_j$ and the periodicity condition are used. These conditions lead to two nonlinear equations for the numerical determination of the quantities a_j and b_j in the case where the penetration of interacting bodies D_j in the *j*-th strip is prescribed. The pressure $p_j(x)$ in the contact region $-a_j < x < b_j$ is calculated in accordance with the relation:

$$p_j(x) = -p_0 e^{(x-a_j)/\beta V} + \frac{E h_j}{2H(l^2 + 4\pi^2 \beta^2 V^2)} \Big[(l^2 + 4\pi^2 \alpha\beta V^2) \left(\cos\frac{2\pi x}{l} - \\ - e^{(x-a_j)/\beta V} \cos\frac{2\pi a}{l} \right) - 2\pi l V (\alpha - \beta) \left(\sin\frac{2\pi x}{l} - e^{(x-a_j)/\beta V} \sin\frac{2\pi a}{l} \right) \\ + 2 D_j l^2 h \left(1 - e^{(x-a_j)/\beta V} \right) \Big] + \frac{E}{2H} (2D_j - h_j) \left(1 - e^{(x-a_j)/\beta V} \right) \tag{19}$$

A similar relation is obtained for the determination of the unknown function of displacement $w_j(x)$ in the region of adhesive interaction $b_j < x < l - a_j$.

The normal and tangential forces acting in a strip in one period of the wavy surface are calculated as:

$$P_j = \Delta \int_{-a_j}^{l-a_j} p_j(x)dx, \quad T_j = \Delta \int_{-a_j}^{l-a_j} \tau_j(x)dx \qquad (20)$$

Discrete contact with saturated adhesive interaction exists under the condition that in the solution obtained, the condition $b_j < l - a_j$ is satisfied. If in the solution obtained we have $b_j \geq l - a_j$, then the case of saturated contact is realized. The other condition of the existence of the discrete contact with saturated adhesive interaction follows from the adopted model of adhesion (5) and (6) – the maximum value of the gap between the surfaces $w_j(x) - f_j(x) - D_j$ should not exceed the prescribed value δ_0, i.e., we have:

$$\max\left(w_j(x) - f_j(x) - D_j\right) \leq \gamma / p_0 \qquad (21)$$

If the function $\mathrm{w_j(x)}$ in the interval $b_j < x < l - a_j$ does not satisfy condition (21), then in the j-th strip the discrete contact with saturated adhesion interaction is not realized. In this case, we should seek the solution for the discrete contact with zones of adhesive interaction and zones of free surface.

4.3. Discrete contact with zones of adhesive interaction

In this case (Fig 3,c), we have three different boundary conditions in three zones of interaction. The differential equation (4) is solved in the contact zone $-a_j < x < b_j$ for the contact pressure $\mathrm{p_j(x)}$ and on the remaining intervals for the displacement $\mathrm{w_j(x)}$. As additional conditions, the conditions of continuity for the pressure $\mathrm{p_j(x)}$ and displacement $\mathrm{w_j(x)}$ at the points $x = -a_{1\mathrm{j}}$, $x = -a_\mathrm{j}$ and $x = b_\mathrm{j}$, $x = b_{1\mathrm{j}}$ and the conditions of periodicity are used. Also, for the determination of the end points of the zones of adhesive interaction, $a_{1\mathrm{j}}$ and $b_{1\mathrm{j}}$, we use the conditions following from (5) and (6), in accordance to which the value of gap between the surfaces at the points $x = -a_{1\mathrm{j}}$ and $x = b_{1\mathrm{j}}$ must be equal to δ_0. These conditions have the form:

$$\begin{aligned} w(-a_1) - f(-a_1) - D &= \gamma / p_0, \\ w(b_1) - f(b_1) - D &= \gamma / p_0 \end{aligned} \qquad (22)$$

As a result, we obtain four nonlinear algebraic equations for numerical determination of the quantities a_j, b_j, $a_{1\mathrm{j}}$, and $b_{1\mathrm{j}}$, provided that the penetration of the bodies in the j-th strip $\mathrm{D_j}$ is prescribed. The pressure $p_\mathrm{j}(\mathrm{x})$ in the contact zone $-a_j < x < b_j$ is specified by relation (19).

Note that the regimes of discrete contact with saturated adhesion and discrete contact with zones of adhesive interaction include also the cases, where there is no contact between the surfaces, and only adhesive interaction occurs over the entire surface or in separate zones of adhesive interaction. The solution for these cases can be easily obtained from Eq. (4) with the second condition of (11) imposed on the entire surface or in a periodical set of zones.

5. RESULTS OF CALCULATIONS

5.1. Solution for a 2-D wavy surface

Below the results of calculations are presented which illustrate the influence of viscoelastic properties, geometrical and adhesive parameters on the contact characteristics and sliding friction force for the case of two-dimensional contact problem. The shape of the wavy surface is described by the function $f(x) = h\, sin^2(\pi x / l)$.

In Fig. 4, the distribution of the normal stress $p(x)$ (curves 1) and tangential stress $\tau(x)$ (curves 2) is shown. Fig. 4,a corresponds to no adhesion and Fig. 4b to the case of adhesion.

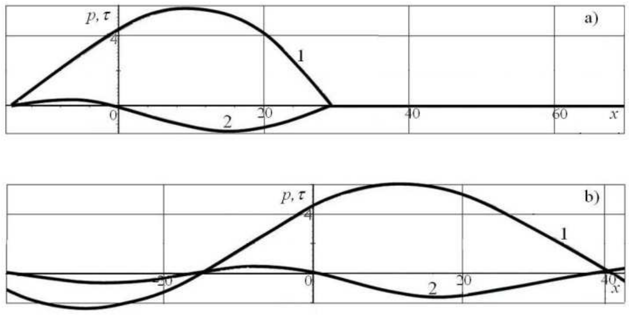

Fig. 4. Distribution of normal and tangential stresses in a period for the case of no adhesion (a) and with adhesion (b)

In Fig. 4, the stresses p and τ are measured in MPa, the x-coordinate in meters. These results are obtained for the material parameters $E/H = 2 \times 10^6$ Pa/m, $T_\sigma = 0{,}003$ s, $T_\varepsilon / T_\sigma = 1000$ load per unit length $P = 154$ N/m, sliding velocity $V = 0{,}1$ m/s, waviness parameters $l = 0{,}086$ mm, $h = 0{,}008$ mm, and adhesion parameters $\gamma = 0{,}05$ N/m, $\mathrm{p_0} = 5{,}5 \times 10^6$ Pa. The friction coefficient calculated for the case without adhesion (a) is $\mu = 0{,}139$, and for the case with adhesion (b) it is $\mu = 0{,}273$. The results indicate that taking into account adhesion not only leads to increase in the friction coefficient, but it also can change the regime of contact and lead from discrete contact (a) to saturated contact (b).

In Fig. 5, the contact width $\mathrm{a + b}$ (a) and the friction force T (b) versus load P are presented for the cases without adhesion (curves 1) and with adhesion (curves 2). The contact width is measured in meters, the forces per unit length T and P in N/m. The results are obtained for the material parameters $E/H = 2 \times 10^6$ Pa/m, $T_\sigma = 0{,}003$ s, $T_\varepsilon / T_\sigma = 1000$, sliding velocity $V = 0{,}1$ m/s, waviness parameters $l = 0{,}1$ mm, $h = 0{,}01$ mm, and adhesion parameters $\gamma = 0{,}01$ N/m, $\mathrm{p_0} = 5 \times 10^3$ Pa. As the load increases the contact width and friction force increase until they attain saturation which means that transition from discrete to saturated contact occurs. The behavior of the contact characteristics (contact width, shift of the contact region with respect to the symmetry axis, contact pressure distribution) differ significantly in the regimes of discrete and saturated contact.

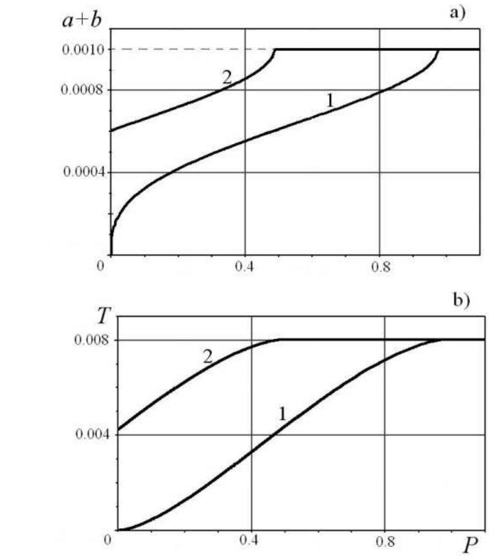

Fig. 5. The contact width (a) and friction force (b) vs normal load for the case with no adhesion (curves 1) and with adhesion (curves 2)

Comparison of curves 1 and 2 shows that taking into account adhesive interaction leads to a considerable increase in the real contact area and hysteretic component of the friction force in the case of discrete contact of the surfaces. Also, taking into account adhesion leads to appearance of negative pressure in the contact region. In the case of saturated contact, adhesion does not influence the contact area and friction force, but it may influence the contact pressure distribution. Results indicate also that with taking into account adhesion, transition from discrete to saturated contact occurs for lower load than without adhesion.

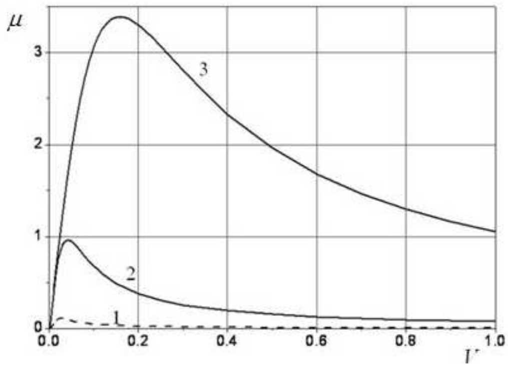

Fig. 6. The friction coefficient vs sliding velocity for different values of the adhesive stress

Figure 6 shows the friction coefficient vs sliding velocity [m/s] for various values of the adhesive stress. Dashed line corresponds to no adhesion (curve 1), curves 2 and 3 correspond to the cases with adhesion. The results are calculated for the mechanical parameters $E/H = 10^9$ Pa/m, $T_\sigma = 0{,}001$ s, $T_\varepsilon/T_\sigma = 10$, roughness parameters $l = 0{,}001$ mm, $h = 0{,}0001$ mm and adhesion parameters $\gamma = 0{,}01$ N/m, $p_0 = 5 \times 10^4$ Pa (curve 2) and $p_0 = 5 \times 10^5$ Pa (curve 3). In the presence of adhesion, the friction coefficient nonmonotonically depends on the velocity and tends to zero for large and small velocities, as it is the case without adhesion. Taking into account adhesion leads to the increase in the value of the friction coefficient, this increase is larger for higher p_0, provided that the surface energy γ is constant.

5.2. Solution for a 3-D wavy surface

Below the result of calculations for a 3D wavy surface (Fig. 1) are presented, the shape of which is described by the function (1).

In Fig. 7, the distributions of contact pressure $p(x,y)$ in the domain $x \in (-l/2; l/2)$; $y \in (0;\ l/2)$ are presented without adhesion (a) and in the presence of adhesion (b) for the same value of the load per one asperity (for one period) $P = 6{,}3561$ H. The results are obtained for $l = 0{,}005$ mm, $h = 0{,}0005$ mm, $E/H = 2 \times 10^9$ Pa/m, $T_\sigma = 0{,}0001$ s, $T_\varepsilon/T_\sigma = 10$, $V = 1$ m/s. Similarly to the 2D case, taking into account the adhesive interaction leads to an increase in the contact areas and, in some conditions, to their merging and passing to the regime of saturated contact (Fig. 2b).

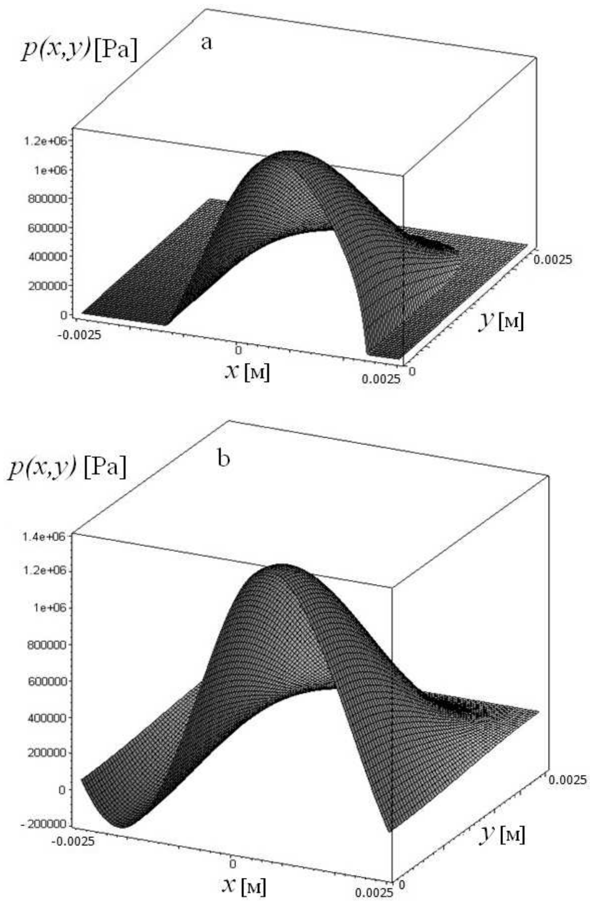

Fig. 7. Contact pressure distributions in a period of waviness without adhesion (a) and with adhesion (b)

The plots of the friction force vs load acting on one asperity are presented in Fig. 8 for the case without adhesion (dashed lines) and with adhesion (solid lines). The mechanical parameters of the material correspond to a kind of rubber. The sliding velocity is $V = 0{,}1$ m/s (Fig. 8a) and $V = 1\ \mathrm{m/s}$ (Fig. 8b). The adhesion parameters are $\gamma = 0{,}01\ \mathrm{N/m}$ and $p_0 = 5{,}5 \times 10^5$ Pa. The waviness parameters are $l = 0{,}010753$ mm and $h = 0{,}000971$ mm. The results show that increase in the friction force due to adhesion is smaller for higher velocity (i.e. for a material with higher effective compliance). The influence of adhesion on the friction force is significant for relatively small loads when the contact is not saturated. Due to adhesion, the friction force T is nonzero for zero load $P = 0$ and in some range of negative loads. Because of this fact, the friction coefficient $\mu = T/P$ becomes very high for very small loads. This allows us to make the conclusion, that for real rough surfaces, the effect of adhesion is especially significant for the asperities which are under small or negative load, and these asperities can contribute significantly into the total friction force.

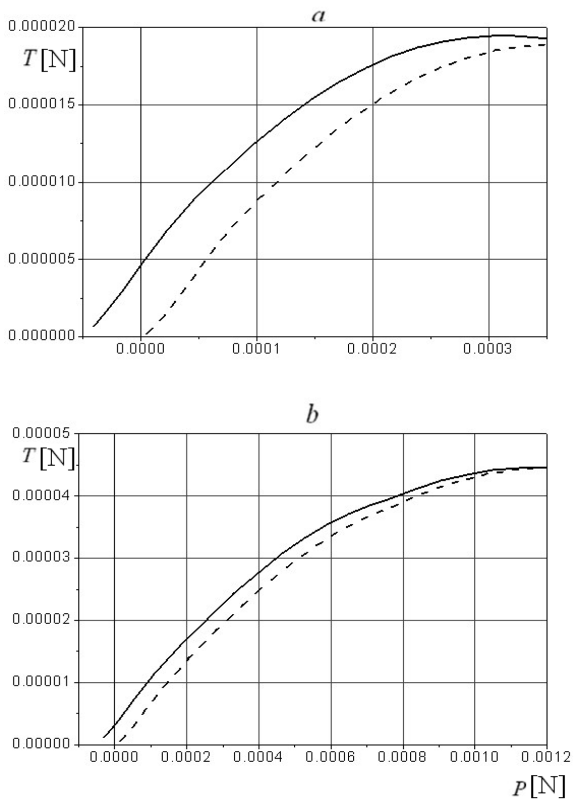

Fig. 8. The friction force vs normal load acting on one asperity with taking into account adhesion (solid lines) and without adhesion (dashed lines)

6. CONCLUSION

A model is suggested to study the adhesion effect on the hysteretic friction force in sliding of rough viscoelastic bodies. The model is based on the solution of a contact problem for a 3-D wavy rigid body sliding on the surface of the viscoelastic foundation taking into account the molecular attraction in the gap between the surfaces.

The results of calutions allow us to draw the following conclusion:

- taking into account the adhesive interaction leads to a significant increase in the real contact area and hysteretic friction forc;
- the transition from discrete to saturated contact in the presence of adhesion occurs at lower loads than without adhesion;
- due to adhesion, the contact between surfaces exists even for negative (tensile) loads;
- the effect of adhesion is especially significant for asperities which are under small or negative load;
- as the adhesion stress increases, the friction force increases, provided that the contact saturation is not attained.

The results obtained can be used for the analysis of the stress-strain state of surface layers and evaluation of the friction force at various scale levels of roughness in sliding of viscoelastic bodies. The mechanisms of fricrtion that were studied play a particular role for micro- and nano-scale levels for whith the size of the gap is close In order of magnitude to the radius of adhesive forces action.

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The work was carried out under the financial support of Russian Foundation for Basic Research (grants No. 11-01-00650 and 11-01-90419)