STABILITY OF THE SECOND FORNASINI-MARCHESINI TYPE MODEL OF CONTINUOUS-DISCRETE LINEAR SYSTEMS

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Abstract: The problem of asymptotic stability of continuous-discrete linear systems is considered. Simple necessary conditions and two computer methods for investigation of asymptotic stability of the second Fornasini-Marchesini type model are given. The first method requires computation of the eigenvalue-loci of complex matrices, the second method requires computation of determinants of some matrices. Effectiveness of the methods is demonstrated on numerical example.

1. INTRODUCTION

In continuous-discrete systems both continuous-time and discrete-time components are relevant and interacting and these components can not be separated. Such systems are called the hybrid systems.

In this paper we consider the continuous-discrete linear systems whose models have structure similar to the models of 2D discrete-time linear systems. Such models, called the 2D continuous-discrete or 2D hybrid models, have been considered in Kaczorek (2002) in the case of positive systems.

The new general model of positive 2D hybrid linear systems has been introduced in Kaczorek (2007) for standard and in Kaczorek (2008a) for fractional systems. The realization, reachability and solvability problems of positive 2D hybrid linear systems have been considered in Kaczorek (2002, 2008b, 2011a), Kaczorek and Rogowski (2010), Kaczorek et al. (2008), Sajewski (2009).

The problems of stability and robust stability of 2D continuous-discrete linear systems have been investigated in Bistritz (2003, 2004), Busłowicz (2010a, b, 2011a, b), Busłowicz and Ruszewski (2011a, b), Guiver and Bose (1981) (see also Chapter 12 in Kaczorek (2011a)) for standard and in Kaczorek (2011a, b), Kaczorek and Sajewski (2011) for positive systems.

The main purpose of this paper is to present computational methods for investigation of asymptotic stability of the second Fornasini-Marchesini type model of continuous-discrete linear systems.

The following notation will be used: $\Re$ – the set of real numbers, $\Re_+ = [0,\infty]$, Z_+ – the set of non-negative integers, $\Re^{n\times m}$ – the set of $n\times m$ real matrices and $\Re^n = \Re^{n\times 1}$, $\| x(\cdot) \|$ – the norm of $x(\cdot)$, $\lambda_i(X)$ – i-th eigenvalue of matrix X.

2. PRELIMINARIES AND FORMULATION OF THE PROBLEM

Consider the state equation of the second Fornasini-Marchesini type model of a continuous-discrete linear system (Kaczorek, 2002) (for $i \in Z_+$ and $t \in \Re_+$):

$$\dot{x}(t,i+1) = A_1\dot{x}(t,i) + A_2 x(t,i+1) + B_1\dot{u}(t,i) + B_2 u(t,i+1), \quad (1)$$

where $\dot{x}(t,i) = \partial x(t,i)/\partial t$, $x(t,i) \in \Re^n$, $u(t,i) \in \Re^m$ and $A_1, A_2 \in \Re^{n\times n}$, $B_1, B_2 \in \Re^{n\times m}$.

Definition 1. The model (1) is called asymptotically stable (or Hurwitz-Schur stable) if for $u(t,i) \equiv 0$ (then also $\dot{u}(t,i) \equiv 0$) and bounded boundary conditions:

$$x(0,i), \; i \geq 1, \; i \in Z_+, \; x(t,0), \; \dot{x}(t,0), \; t \in \Re_+, \quad (2)$$

the following condition holds:

$$\lim_{i,t\to\infty} \| x(t,i) \| = 0 \text{ for } t,i \to \infty.$$

The characteristic matrix of the model (1) has the form:

$$H(s,z) = szI_n - sA_1 - zA_2. \quad (3)$$

The characteristic function of this model:

$$w(s,z) = \det H(s,z) = \det[szI_n - sA_1 - zA_2] \quad (4)$$

is a polynomial in two independent variables s and z, of the general form:

$$w(s,z) = \sum_{k=0}^{n}\sum_{j=0}^{n} a_{kj} s^k z^j, \quad a_{nn} = 1. \quad (5)$$

From Bistritz (2003, 2004) and Guiver and Bose (1981) we have the following theorem.

Theorem 1. The model (1) with characteristic function (4) is asymptotically stable if and only if the following condition holds:

$$w(s,z) \neq 0, \quad \operatorname{Re} s \geq 0, \quad |z| \geq 1. \quad (6)$$

The polynomial $w(s,z)$ satisfying the condition (6) is called continuous-discrete stable (C-D stable) or Hurwitz-Schur stable (Bistritz (2003, 2004)).

The main purpose of this paper is to present computational methods for checking the condition (6) of asymptotic stability of the model (1) of continuous-discrete linear systems.

3. SOLUTION OF THE PROBLEM

Lemma 1. Simple necessary conditions for asymptotic stability of the model (1) are as follows:

$$\text{Re}\,\lambda_i(A_2) < 0, \quad i = 1,2,...,n, \tag{7}$$

$$|\lambda_i(A_1)| < 1, \quad i = 1,2,...,n. \tag{8}$$

Proof. From (1) for $A_1 \equiv 0$ and $B_1 = B_2 \equiv 0$ one obtains the homogeneous state equation of the continuous-time linear system (for the fixed $i \in Z_+$):

$$\dot{x}(t,i+1) = A_2 x(t,i+1). \tag{9}$$

The system (9) is asymptotically stable if and only if the condition (7) holds, i.e. the matrix A_2 is Hurwitz stable (is a Hurwitz matrix).

Similarly, substitution of $A_2 \equiv 0$ and $B_1 = B_2 \equiv 0$ in (1) gives the equation:

$$\dot{x}(t,i+1) = A_1 \dot{x}(t,i), \tag{10}$$

which can be written in the form:

$$v(t,i+1) = A_1 v(t,i), \quad v(t,i) = \dot{x}(t,i). \tag{11}$$

The discrete-time linear system (11) is asymptotically stable if and only if the condition (8) holds, i.e. the matrix A_1 is Schur stable (is a Schur matrix). This completes the proof.

Theorem 2. The condition (6) is equivalent to the following two conditions:

$$w(s, e^{j\omega}) \neq 0, \quad \text{Re}\,s \geq 0, \quad \forall \omega \in \Omega = [0, 2\pi], \tag{12}$$

and

$$w(jy, z) \neq 0, \quad |z| \geq 1, \quad \forall y \in [0, \infty). \tag{13}$$

Proof. From Bistritz (2003, 2004) and Guiver and Bose (1981) it follows that (6) is equivalent to the conditions:

$$w(s,z) \neq 0, \quad \text{Re}\,s \geq 0, \quad |z| = 1, \tag{14}$$

and

$$w(s,z) \neq 0, \quad \text{Re}\,s = 0, \quad |z| \geq 1. \tag{15}$$

It is easy to see that conditions (14) and (15) can be written in the forms (12) and (13), respectively.

The characteristic matrix (3) of the model (1) can be written in the following forms:

$$H(s,z) = [zI - A_1][sI - S_1(z)] = [sI - A_2][zI - S_2(s)], \tag{16}$$

where:

$$S_1(z) = (zI - A_1)^{-1}(zA_2), \tag{17}$$

$$S_2(s) = (sI - A_2)^{-1}(sA_1). \tag{18}$$

Hence,

$$w(s,z) = \det[zI - A_1]\det[sI - S_1(z)], \tag{19a}$$

$$w(s,z) = \det[sI - A_2]\det[zI - S_2(s)]. \tag{19b}$$

From (19a) and (17) for $z = e^{j\omega}$ we have:

$$w(s, e^{j\omega}) = \det[Ie^{j\omega} - A_1]\,\det[sI - S_1(e^{j\omega})], \tag{20}$$

where:

$$S_1(e^{j\omega}) = (Ie^{j\omega} - A_1)^{-1} A_2 e^{j\omega}. \tag{21}$$

Lemma 2. Let the necessary condition (8) be satisfied. The condition (12) holds if and only if all eigenvalues of the complex matrix (21) have negative real parts for all $\omega \in [0, 2\pi]$.

Proof. If (8) is satisfied then the matrix $Ie^{j\omega} - A_1$ is nonsingular for all $\omega \in [0, 2\pi]$ and from (20) it follows that the condition (12) holds if and only if:

$$\det(sI - S_1(e^{j\omega})) \neq 0, \quad \text{Re}\,s \geq 0, \quad \forall \omega \in \Omega = [0, 2\pi]. \tag{22}$$

Satisfaction of (22) means that all eigenvalues of (21) have negative real parts for all $\omega \in \Omega$.

The condition (22) holds if and only if the eigenvalue-loci of (21) with $\omega \in \Omega$ are located in the open left half-plane of the complex s plane. These eigenvalue-loci are the closed curves with endpoints (for $\omega = 0$ and $\omega = 2\pi$) in eigenvalues of $S_1(1) = (I - A_1)^{-1} A_2$.

From (19b) and (18) for $s = jy$ we have:

$$w(jy, z) = \det[jyI - A_2]\det[zI - S_2(jy)], \tag{23}$$

where:

$$S_2(jy) = (jyI - A_2)^{-1}(jyA_1). \tag{24}$$

Lemma 3. Let the necessary condition (7) be satisfied. The condition (13) holds if and only if all eigenvalues of the complex matrix (24) have absolute values less than one for all $y \geq 0$.

Proof. If (7) is satisfied then the matrix $jyI - A_2$ is nonsingular for all $y \geq 0$ and from (23) we have that the condition (13) holds if and only if:

$$\det[zI - S_2(jy)] \neq 0, \quad |z| \geq 1, \quad \forall y \in [0, \infty), \tag{25}$$

i.e. all eigenvalues of (24) have absolute values less than one for all $y \geq 0$.

Satisfaction of (25) means that the eigenvalue-loci of (24) (eigenvalues of (24) for all $y \in [0, \infty)$) are located in the open unit circle of the complex z plane.

It is easy to check that:

$$\lim_{y \to \infty} S_2(jy) = \lim_{y \to \infty} \frac{jy \cdot \text{adj}(jyI - A_2)}{\det(jyI - A_2)} A_1 = A_1, \tag{26}$$

where adj(.) denotes the adjoint matrix.

From the above and (24) it follows that the eigenvalue-loci of $S_2(jy)$ start for $y = 0$ in the origin of the complex plane and tend to the eigenvalues of A_1 for $y \to \infty$.

Theorem 3. The second Fornasini-Marchesini type model (1) is asymptotically stable if and only if the necessary conditions (7) and (8) are satisfied and the following conditions hold:

$$\mathrm{Re}\lambda_i\{S_1(e^{j\omega})\} < 0, \quad \forall\omega \in \Omega = [0, 2\pi], \quad i = 1,2,...,n, \tag{27}$$

and

$$|\lambda_i\{S_2(jy)\}| < 1, \quad \forall y \geq 0, \quad i = 1,2,...,n, \tag{28}$$

where the matrices $S_2(e^{j\omega})$ and $S_2(jy)$ have the forms (21) and (24), respectively.

Proof. It follows from Theorem 2 and Lemmas 1, 2 and 3.

Application of Theorem 3 requires computation of eigenvalues of complex matrices (21) and (24). This may be inconvenient from the computational reasons, particularly in the case of ill conditioned matrices.

Therefore, we present a new method for investigation of asymptotic stability of the model (1) which requires the computation of determinants of some matrices.

Consider the polynomial:

$$w_1(s, e^{j\omega}) = \det(sI - S_1(e^{j\omega})), \tag{29}$$

where $S_1(e^{j\omega})$ is defined by (21). From the classical Mikhailov theorem (see Busłowicz (2007), Keel and Bhattacharyya (2000)) it follows that the condition (27) holds if and only if for any fixed $\omega \in [0, 2\pi]$ plot of $w_1(jy, e^{j\omega})$ starts for $y = 0$ in the point $w_1(j0, e^{j\omega}) = \det(-S_1(e^{j\omega}))$ and runs in the positive direction by n quadrants of the complex plane (missing the origin of this plane) if y increases from 0 to $+\infty$. This plot (called the Mikhailov hodograph) quickly tends to infinity as y grows to ∞. Therefore, direct application of the Mikhailov theorem to checking the condition (27) is not practically reliable.

To remove this difficulty, we introduce the rational function:

$$\phi_1(jy, e^{j\omega}) = \frac{w_1(jy, e^{j\omega})}{w_{10}(jy)}, \quad \omega \in \Omega = [0, 2\pi], \tag{30}$$

instead of $w_1(jy, e^{j\omega})$, where $w_{10}(s)$ is any Hurwitz stable reference polynomial of degree n.

Lemma 4. The condition (27) holds if and only if for all fixed $y \geq 0$ plot of (30) does not encircle or cross the origin of the complex plane.

Proof. If the reference polynomial $w_{10}(s)$ is Hurwitz stable then from the Argument Principle we have:

$$\Delta \arg_{y \in (-\infty, \infty)} w_{10}(jy) = n\pi.$$

From (30) it follows that for any fixed $\omega \in \Omega$:

$$\begin{aligned} &\underset{y\in(-\infty,\infty)}{\Delta \arg}\ \phi_1(jy, e^{j\omega}) = \\ &= \underset{y\in(-\infty,\infty)}{\Delta \arg}\ w_1(jy, e^{j\omega}) - \underset{y\in(-\infty,\infty)}{\Delta \arg}\ w_{10}(jy). \end{aligned} \tag{31}$$

The condition (27) holds for any fixed $\omega \in \Omega$ if and only if:

$$\underset{y\in(-\infty,\infty)}{\Delta \arg}\ w_1(jy, e^{j\omega}) = \underset{y\in(-\infty,\infty)}{\Delta \arg}\ w_{10}(jy) = n\pi, \tag{32}$$

which holds if and only if $\Delta\arg_{y\in(-\infty,\infty)}\phi(jy, e^{j\omega}) = 0$.

Taking into account all $\omega \in \Omega$, we obtain that the above holds if and only if for all fixed $y \geq 0$ plot of (30) as a function of $\omega \in \Omega$ does not encircle or cross the origin of the complex plane.

The reference polynomial $w_{10}(s)$ can be chosen in the form:

$$w_1(s,1) = \det(sI - S_1(1)), \quad S_1(1) = (I - A_1)^{-1}A_2. \tag{33}$$

Hurwitz stability of the polynomial (33) is necessary for Hurwitz stability of the complex polynomial (29) for all $\omega \in \Omega$:

If $w_{10}(s) = w_1(s,1)$ then:

$$\phi_1(jy, e^{j\omega}) = \frac{w_1(jy, e^{j\omega})}{w_1(jy,1)}, \quad \omega \in \Omega. \tag{34}$$

Plot of (34) as a function of $\omega \in \Omega$ (with any fixed $y \geq 0$) is a closed curve. It begins with $\omega = 0$ and ends with $\omega = 2\pi$ in the point $(1, j0)$, because $\phi_1(jy, 1) = 1$. It is easy to check that if $y \to \infty$, then the closed curve (34) reduces to the point $(1, j0)$.

The plot of (34) is called the modified Mikhailov hodograph. From the above it follows that this hodograph is bounded for all $y \geq 0$.

Now, we consider the complex polynomial:

$$w_2(jy, z) = \det(zI - S_2(jy)), \tag{35}$$

where $S_2(jy)$ is defined by (24).

Let $w_{20}(z)$ be any Schur stable reference polynomial of degree n.

Similarly as for Lemma 4, we obtain the following lemma.

Lemma 5. The condition (28) holds if and only if for all fixed $y \geq 0$ plot of the function:

$$\phi_2(jy, e^{j\omega}) = \frac{w_2(jy, e^{j\omega})}{w_{20}(e^{j\omega})}, \quad \omega \in \Omega, \tag{36}$$

does not encircle or cross the origin of the complex plane, where $w_2(jy, e^{j\omega})$ has the form (35) for $z = e^{j\omega}$.

From (35) for $y = 0$ we have:

$$w_2(0, z) = \det(zI - S_2(0)) = z^n.$$

Therefore, the reference polynomial $w_{20}(z)$ can be chosen as $w_{20}(z) = w_2(0,\ z) = z^n$.

Schur stability of $w_2(0,\ z)$ is necessary for Schur stability of the complex polynomial (35) for all $y \geq 0$.

If $w_{20}(z) = z^n$ then:

$$\phi_2(jy, e^{j\omega}) = \frac{w_2(jy, e^{j\omega})}{e^{j\omega n}}, \quad \omega \in \Omega. \tag{37}$$

Plot of (37) as a function of $\omega \in \Omega$ with the fixed $y \geq 0$ is a closed curve. It begins with $\omega = 0$ and ends with $\omega = 2\pi$ in the point:

$$\phi_2(jy,1) = w_2(jy,1) = \det(I - S_2(jy)). \quad (38)$$

It is easy to see that $\phi_2(0,1) = 1$.

From (26) and (37) we have:

$$\phi_2(\infty, e^{j\omega}) = \lim_{y\to\infty} \phi_2(jy, e^{j\omega}) = \frac{\det(e^{j\omega}I - A_1)}{e^{j\omega n}}, \; \omega \in \Omega. \quad (39)$$

From the above it follows that if $y \to \infty$ then plot of (37) tends to the closed curve (39) with endpoints (for $\omega = 0$ and $\omega = 2\pi$) $\phi_2(\infty, 1) = \det(I - A_1)$.

From Theorem 3 and Lemmas 4 and 5 we have the following theorem.

Theorem 4. Assume that the necessary conditions (7) and (8) are satisfied. The model (1) is asymptotically stable if and only if the following two conditions hold:

1. plots of the function (34) do not encircle or cross the origin of the complex plane for all fixed $y \geq 0$;
2. plots of the function (37) do not encircle or cross the origin of the complex plane for all fixed $y \geq 0$.

Applying computational method given in Theorem 4 we can take into consideration the following remark.

Remark. The range $Y = [0, y_f]$ of values of the parameter y should be a suitable large, such that from plots of the functions (34) and (37) for $y \in Y$ we can affirm fulfilment (or not) the conditions of Theorem 4 for all $y \geq 0$. For any fixed $y \in Y$ determined with appropriately small step Δy, plots of the functions (34) and (37) should be draw separately digitizing the range $\Omega = [0, 2\pi]$ with a sufficiently small step $\Delta\omega$.

4. ILLUSTRATIVE EXAMPLE

Consider the second Fornasini-Marchesini type model (1) with the matrices:

$$A_1 = \begin{bmatrix} 0.4 & 0.1 & 0.3 \\ 0 & -0.8 & -0.4 \\ -0.3 & 0.2 & 0.4 \end{bmatrix}, \; A_2 = \begin{bmatrix} -1.4 & -1 & -0.1 \\ 0.8 & -0.4 & 0 \\ -1 & 0 & -0.7 \end{bmatrix}. \quad (40)$$

Computing eigenvalues of A_1 and A_2 one obtains:

– eigenvalues of A_1:

$$z_1 = -0.7247; \quad z_{2,3} = 0.3624 \pm j0.2897$$

– eigenvalues of A_2:

$$s_1 = -0.7640; \quad s_{2,3} = -0.8680 \pm j0.6635.$$

From the above it follows that the necessary conditions (7) and (8) hold.

Eigenvalue-loci of the matrices $S_1(e^{j\omega})$, $\omega \in [0, 2\pi]$ and $S_2(jy)$, $y \in [0, 50]$ are shown in Fig. 1 and 2, respectively. By 'o' in Fig. 2 are denoted points corresponding to eigenvalues of A_1. The eigenvalue-loci of $S_2(jy)$ tend to these points if $y \to \infty$.

From Fig. 1 and 2 it follows that the conditions (27) and (28) of Theorem 3 are satisfied and the system is asymptotically stable.

Plots of the functions (34) for $y \in [0, 40]$ and (37) for $y \in [0, 20]$ are shown in Fig. 3 and 4, respectively. The curve (39) is denoted by stars in Fig. 4.

From Fig. 3 and 4 it follows that the conditions of Theorem 4 are satisfied and the system is asymptotically stable.

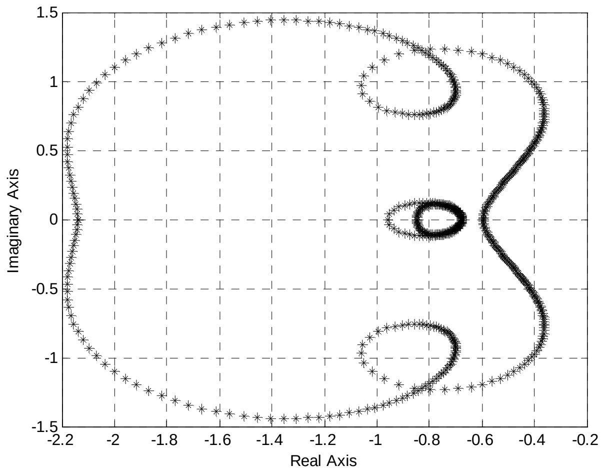

Fig. 1. Eigenvalue-loci of $S_1(e^{j\omega})$, $\omega \in \Omega = [0, 2\pi]$

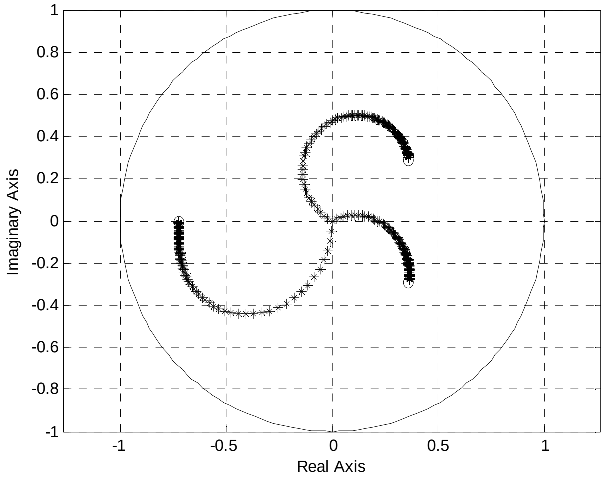

Fig. 2. Eigenvalue-loci of $S_2(jy)$, $y \in [0, 50]$

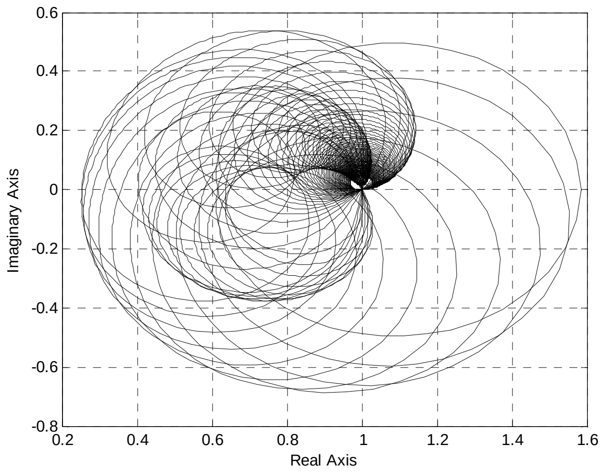

Fig. 3. Plot of (34) for $y \in [0, 40]$

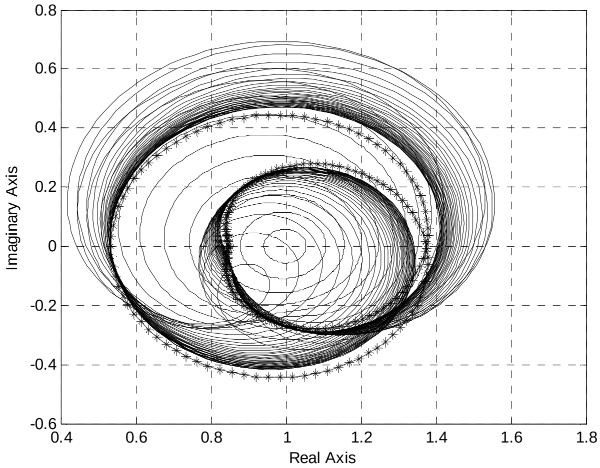

Fig. 4. Plot of (37) for $y \in [0, 20]$

5. CONCLUDING REMARKS

Simple necessary conditions and computational methods for analysis of asymptotic stability of the second Fornasini-Marchesini type model (1) of continuous-discrete linear systems have been given in Lemma 1 and Theorems 3 and 4, respectively. The method given in Theorem 3 requires computation of eigenvalue-loci of complex matrices (21) and (24). The method proposed in Theorem 4 requires computation of values of complex functions (34) and (37).

The method of Theorem 3 has been generalized in Busłowicz (2011b) for the first Fornasini-Marchesini type and the Roesser type models of continuous-discrete linear systems. The method of Theorem 4 has been applied in Busłowicz and Ruszewski (2011a) to asymptotic stability analysis of the first Fornasini-Marchesini type model.

Extension of the proposed methods for the new general type model of 2D continuous-discrete linear systems has been given in Busłowicz and Ruszewski (2011b).

REFERENCES

1. **Bistritz Y.** (2003), A stability test for continuous-discrete bivariate polynomials, *Proc. Int. Symp. on Circuits and Systems*, vol. 3, 682-685, Bangkok, Thailand.
2. **Bistritz Y.** (2004), Immittance and telepolation-based procedures to test stability of continuous-discrete bivariate polynomials, *Proc. IEEE Int. Symposium on Circuits and Systems*, vol. 3, 293-296, Vancouver, Canada.
3. **Busłowicz M.** (2007), *Stability of linear time-invariant systems with uncertain parameters*, Publishing Department of Technical University of Białystok, Białystok (in Polish)
4. **Busłowicz M.** (2010a), Robust stability of the new general 2D model of a class of continuous-discrete linear systems, *Bull. Pol. Ac. Techn. Sci.*, Vol. 57, No. 4, 561-565.
5. **Busłowicz M.** (2010b), Stability and robust stability conditions for general model of scalar continuous-discrete linear systems, *Measurement Automation and Monitoring*, Vol. 56, No. 2, 133-135.
6. **Busłowicz M.** (2011a), Improved stability and robust stability conditions for general model of scalar continuous-discrete linear systems, *Measurement Automation and Monitoring*, Vol. 57, No. 2, 188-189.
7. **Busłowicz M.** (2011b), Computational methods for investigation of stability of models of 2D continuous-discrete linear systems, *Journal of Automation, Mobile Robotics and Intelligent Systems*, Vol. 5, No. 1, 3-7.
8. **Busłowicz M.**, **Ruszewski A.** (2011a), Stability investigation of continuous-discrete linear systems, *Measurement Automation and Robotics*, No.2/2011, 566-575 (in Polish).
9. **Busłowicz M.**, **Ruszewski A.** (2011b), Computer methods for stability analysis of the Roesser type model of 2D continuous-discrete linear systems, *Int. J. Appl. Math. Comput. Sci.* (in press).
10. **Guiver J. P., Bose N. K.** (1981), On test for zero-sets of multivariate polynomials in noncompact polydomains, *Proc. of the IEEE*, Vol. 69, No. 4, 467-469.
11. **Kaczorek T.** (2002), *Positive 1D and 2D Systems*, Springer-Verlag, London.
12. **Kaczorek T.** (2007), Positive 2D hybrid linear systems, *Bull. Pol. Ac. Techn. Sci.*, Vol. 55, No. 4, 351-358.
13. **Kaczorek T.** (2008a), Positive fractional 2D hybrid linear systems, *Bull. Pol. Ac. Techn. Sci.*, Vol. 56, No. 3, 273-277.
14. **Kaczorek T.** (2008b), Realization problem for positive 2D hybrid systems, *COMPEL*, Vol. 27, No. 3, 613-623.
15. **Kaczorek T.** (2011a), *Selected Problems of Fractional Systems Theory*, Springer-Verlag, Berlin.
16. **Kaczorek T.** (2011b), New stability conditions for positive continuous-discrete 2D linear systems. *Int. J. Appl. Math. Comput. Sci.*, Vol. 21, No. 3, 521-524.
17. **Kaczorek T.**, **Rogowski K.** (2010), Reachability of linear hybrid systems described by the general model, *Archives of Control Sciences*, Vol. 20, No. 2, 199-2007.
18. **Kaczorek T. and Sajewski Ł.** (2011), Stability of continuous-discrete linear systems with delays in state vector, *Archives of Control Sciences*, Vol. 21, No. 1, 5-16.
19. **Kaczorek T., Marchenko V., Sajewski Ł.** (2008), Solvability of 2D hybrid linear systems - comparison of the different methods, *Acta Mechanica et Automatica*, Vol. 2, No. 2, 59-66.
20. **Keel L. H., Bhattacharyya S. P.** (2000), A generalization of Mikhailov's criterion with applications, *Proc. of American Control Conference*, Chicago, Vol. 6, 4311-4315.
21. **Sajewski Ł.** (2009), Solution of 2D singular hybrid linear systems, *Kybernetes*, Vol. 38, No. 7/8, 1079-1092.

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