ABSTRACTS

Marek Błasik, Małgorzata Klimek
On Application of the Contraction Principle to Solve the Two-Term Fractional Differential Equations

We solve two-term fractional differential equations with left-sided Caputo derivatives. Existence-uniqueness theorems are proved using newly-introduced equivalent norms/metric on the space of continuous functions. The metrics are modified in such a way that the space of continuous functions is complete and the Banach theorem on a fixed point can be applied. It appears that the general solution is generated by the stationary function of the highest order derivative and exists in an arbitrary interval [0,b]

Tomasz Błaszczyk, Ewa Kotela, Matthew R. Hall, Jacek Leszczyński
Analysis and Applications of Composed Forms of Caputo Fractional Derivatives

In this paper we consider two ordinary fractional differential equations with composition of the left and the right Caputo derivatives. Analytical solution of this type of equations is known for particular cases, having a complex form, and therefore is difficult in practical calculations. Here, we present two numerical schemes being dependent on a fractional order of equation. The results of numerical calculations are compared with analytical solutions and then we illustrate convergence of our schemes. Finally, we show an application of the considered equation.

Mikołaj Busłowicz
Stability of State-Space Models of Linear Continuous-Time Fractional Order Systems

The paper considers the stability problem of linear time-invariant continuous-time systems of fractional order, standard and positive, described by the state space model. Review of previous results is given and some new methods for stability checking are presented. Considerations are illustrated by numerical examples and results of computer simulations.

Stefan Domek
Fuzzy Predictive Control of Fractional-Order Nonlinear Discrete-Time Systems

At the end of the 19th century Liouville and Riemann introduced the notion of a fractional-order derivative, and in the latter half of the 20th century the concept of the so-called Grünewald-Letnikov fractional-order discrete difference has been put forward. In the paper a predictive controller for MIMO fractional-order discrete-time systems is proposed, and then the concept is extended to nonlinear processes that can be modelled by Takagi-Sugeno fuzzy models. At first nonlinear and linear fractional-order discrete-time dynamical models are described. Then a generalized nonlinear fractional-order TS fuzzy model is defined, for which equations of a predictive controller are derived.

Marcin Graba
The Influence of Geometry of the Specimen and Material Properties on the Q-Stress Value Near the Crack Tip for SEN(T) Specimen

In the paper the short theoretical backgrounds about elastic-plastic fracture mechanics were presented and the O'Dowd-Shih theory was discussed. Using ADINA System program, the values of the Q-stress determined for various elastic-plastic materials for SEN(T) specimen – single edge notched plates in tension – were presented. The influence of kind of the specimen, crack length and material properties (work-hardening exponent and yield stress) on the Q-parameter were tested. The numerical results were approximated by the closed form formulas. Presented in the paper results are complementary of the two papers published in 2007 (Graba, 2007) and in 2010 (Graba, 2010), which show and describe influence of the material properties and crack length for the Q-stress value for SEN(B) and CC(T) specimens respectively. Presented and mentioned papers show such catalogue of the Q-stress value, which may be used in engineering analysis for calculation of the real fracture toughness.

Piotr Grześ
Partition of Heat in 2D Finite Element Model of a Disc Brake

In this paper nine of formulas (theoretical and experimental) for the heat partition ratio were employed to study the temperature distributions of two different geometrical types of the solid disc brake during emergency brake application. A two-dimensional finite element analysis incorporating specific values of the heat partition ratios was carried out. The boundary heat flux uniformly distributed over the circumference of a rubbing path to simulate the heat generated at the pad/disc interface was applied to the model. A number of factors over the heat partition ratio that affects the temperature fields are included and their importance is discussed.

Tadeusz Kaczorek
Positivity and Reachability of Fractional Electrical Circuits

Conditions for the positivity of fractional linear electrical circuits composed of resistors, coils, condensators and voltage (current) sources are established. It is shown that: 1) the fractional electrical circuit composed of resistors, coils and voltage source is positive for any values of their resistances, inductances and source voltages if and only if the number of coils is less or equal to the number of its linearly independent meshes, 2) the fractional electrical circuit is not positive for any values of its resistances, capacitances and source voltages if each its branch contains resistor, capacitor and voltage source, It is also shown that the fractional positive electrical circuits of *R*, *C*, *e* type are reachable if and only if the conductances between their nodes are zero and the fractional positive electrical circuits of *R*, *L*, *e* type are reachable if and only if the resistances belonging to two meshes are zero.

Tadeusz Kaczorek
Necessary and Sufficient Stability Conditions of Fractional Positive Continuous-Time Linear Systems

Necessary and sufficient conditions for the asymptotic stability of fractional positive continuous-time linear systems are established. It is shown that the matrix A of the stable fractional positive system has not eigenvalues in the part of stability region located in the right half of the complex plane.

Jerzy Klamka
Local Controllability of Fractional Discrete-Time Semilinear Systems

In the paper unconstrained local controllability problem of finite-dimensional fractional discrete-time semilinear systems with constant coefficients is addressed. Using general formula of solution of difference state equation sufficient condition for local unconstrained controllability in a given number of steps is formulated and proved. Simple illustrative example is also presented.

Zbigniew Kulesza
FPGA Based Active Magnetic Bearings Controller

The article discusses main problems of implementing the PID control law in the FPGA integrated circuit. Consecutive steps of discretizing and choosing the fixed-point representation of the continuous, floating-point PID algorithm are described. The FPGA controller is going to be used in the active hetero-polar magnetic bearings system consisting of two radial and one axial bearings. The results of the experimental tests of the controller are presented. The dynamic performance of the controller is better when compared with the dSPACE controller, that was used so far. The designed hardware and software, the developed implementation procedure and the experience acquired during this stage of the whole project are going to be used during the implementation of more sophisticated control laws (e.g. robust) in the FPGA for AMB controllers.

Wojciech Mitkowski
Approximation of Fractional Diffusion-Wave Equation

In this paper we consider the solution of the fractional differential equations. In particular, we consider the numerical solution of the fractional one dimensional diffusion-wave equation. Some improvements of computational algorithms are suggested. The considerations have been illustrated by examples.

Dorota Mozyrska, Ewa Pawłuszewicz
Linear q-Difference Fractional-Order Control Systems with Finite Memory

The formula for the solution to linear q-difference fractional-order control systems with finite memory is derived. New definitions of observability and controllability are proposed by using the concept of extended initial conditions. The rank condition for observability is established and the control law is stated.

Zbigniew Oksiuta
Microstructural Changes of Ods Ferritic Steel Powders During Mechanical Alloying

The ODS ferritic steel powder with chemical composition of Fe-14Cr-2W-0.3Ti-0.3Y2O3 was mechanically alloyed (MA) either from elemental or pre-alloyed powders in a planetary ball mill. Different milling parameters have been used to investigate their influence on the morphology and microstructure of the ODS ferritic steel powder. The time of MA was optimized by studying the structural evolution of the powder by means of X-ray diffractometry and TEM. In the case of elemental powder very small, about 10 µm in diameter, spherical particles with a large surface area have been obtained. Flakey-like particles with an average size of about 45 µm were obtained in the case of the pre-alloyed powder. The lattice strain calculated from XRD spectra of the elemental and pre-alloyed powders exhibits a value of about 0.51 % and 0.67, respectively. The pre-alloyed powder after consolidation process showed the highest density and microhardness value.

Piotr Ostalczyk
Variable-, Fractional-Orders Closed-Loop Systems Description

In this paper we explore the linear difference equations with fractional orders, which are functions of time. A description of closed-loop dynamical systems described by such equations is proposed. In a numerical example a simple control strategy based on time-varying fractional orders is presented.

Piotr Ostalczyk, Dariusz Brzeziński
Numerical Evaluation of Fractional Differ-Integral of Some Elementary Functions via Inverse Transformation

This paper presents methods of calculating fractional differ-integrals numerically. We discuss extensively the pros and cons of applying the Riemann-Liouville formula, as well as direct approach in form of The Grünwald-Letnikov method. We take closer look at the singularity, which appears when using classical form of Riemann-Liouville formula. To calculate Riemann-Liouville differ-integral we use very well-known techniques like The Newton-Cotes Midpoint Rule. We also use two Gauss formulas. By implementing transformation of the core integrand of Riemann-Liouville formula (we called it “the inverse transformation”), we would like to point the possible way of reducing errors when calculating it. The core of this paper is the subject of reducing the absolute error when calculating Riemann-Liouville differ-integrals of some elementary functions; we use our own C++ programs to calculate them and compare obtained results of all methods with, where possible, exact values, where not – with values obtained using excellent method of integration incorporated in Mathematica. We will not discuss complexity of numerical calculations. We will focus solely on minimization of the absolute errors.

Ivo Petráš, Dagmar Bednárová
Control of Fractional-Order Nonlinear Systems: A Review

This paper deals with the control of the fractional-order nonlinear systems. A list of the control strategies as well as synchronization of the chaotic systems is presented. An illustrative example of sliding mode control (SMC) of the fractional-order (commensurate and incommensurate) financial system is described and commented together with the simulation results.

Paweł Piątek, Jerzy Baranowski
Investigation of Fixed-Point Computation Influence on Numerical Solutions of Fractional Differential Equations

In this paper the problem of the influence of fixed point computation on numerical solutions of linear differential equations of fractional order is considered. It is a practically important problem, because of potential possibilities of using dynamical systems of fractional order in the tasks of control and filtering. Discussion includes numerical method is based on the Grünwald-Letnikov fractional derivative and how the application of fixed-point architecture influences its operation. Conclusions are illustrated with results of floating-point arithmetic with double precision and fixed point arithmetic with different word lengths.

Yuriy Povstenko
Solutions to Time-Fractional Diffusion-Wave Equation in Spherical Coordinates

Solutions to time-fractional diffusion-wave equation with a source term in spherical coordinates are obtained for an infinite medium. The solutions are found using the Laplace transform with respect to time t, the finite Fourier transform with respect to the angular coordinate φ, the Legendre transform with respect to the spatial coordinate μ, and the Hankel transform of the order n+1/2 with respect to the radial coordinate r. In the central symmetric case with one spatial coordinate r the obtained results coincide with those studied earlier.

Krzysztof Rogowski
General Response Formula for Fractional 2D Continuous-Time Linear Systems Described by the Roesser Model

A new class of fractional two-dimensional (2D) continuous-time linear systems is introduced. The general response formula for the system is derived using a 2D Laplace transform. It is shown that the classical Cayley-Hamilton theorem is valid for such class of systems. Usefulness of the general response formula to obtain a solution of the system is discussed and illustrated by a numerical example.

Andrzej Ruszewski, Tomasz Nartowicz
Stabilization of Inertial Plant with Time Delay Using Fractional Order Controller

The paper presents the problem of designing of a fractional order controller satisfying the conditions of gain and phase margins of the closed-loop system with time-delay inertial plant. The transfer function of the controller follows directly from the use of Bode's ideal transfer function as a reference transfer function for the open loop system. Using the classical D-partition method and the gain-phase margin tester, a simple computational method for determining stability regions in the controller parameters plane is given. An efficient analytical procedure to obtain controller parameter values for specified gain and phase margin requirements is also given. The considerations are illustrated by numerical examples computed in MATLAB/Simulink.

Łukasz Sajewski
Positive Realization of SISO 2D Different Orders Fractional Discrete-Time Linear Systems

The realization problem for single-input single-output 2D positive fractional systems with different orders is formulated and a method based on the state variable diagram for finding a positive realization of a given proper transfer function is proposed. Sufficient conditions for the existence of a positive realization of this class of 2D linear systems are established. A procedure for computation of a positive realization is proposed and illustrated by a numerical example.

Dominik Sierociuk, Grzegorz Sarwas, Andrzej Dzieliński
Discrete Fractional Order Artificial Neural Network

In this paper the discrete time fractional order artificial neural network is presented. This structure is proposed for simulating the dynamics of non-linear fractional order systems. In the second part of this paper several numerical examples are shown. The final part of the paper presents the discussion on the use of fractional or integer discrete time neural network for modelling and simulating fractional order non-linear systems. The simulation results show the advantages of the proposed solution over the classical (integer) neural network approach to modelling of non-linear fractional order systems.